ME 307 – Machine Elements I

Chapter 2

Stress Analysis (Part I)

Mechanical Engineering
University of Gaziantep

Dr. Sadık Olguner
Dr. Hakan Çandar

- What is stress?
- What is the difference between stress and pressure?

$$\sigma = stress \qquad P = pressure$$

$$\sigma = \frac{F}{A} \qquad P = \frac{F}{A}$$

$$\sigma = \frac{N}{mm^2} = MPa \qquad P = \frac{N}{mm^2} = MPa$$

- Can we say $\boldsymbol{\sigma = P}$?

Absolutely No!

- When a load / loads are applied to an element, the element **tries to resist** for any deformation. This internal resistance to the external load is stress.
- Being an internal resistance, **stress can not be measured**, only can be calculated by
  - **Analytically** (by using stress formulations)
  - **FEM analysis** (ANSYS, DEFORM, Abaqus)
  - **Strain gauges** (by measuring strain)
- On the other hand, pressure is an external effect, and **can be directly measured by pressure gauges**.
- Therefore, the main difference between stress and pressure is, stress is an internal resistance and can not be directly measured, but the pressure is an external effect and can be measured.

Stress

Normal Stress

- Normal stress is the stress acting normal to the stress element.
- Changes both;
  - Volume
  - Shapes of the stress element

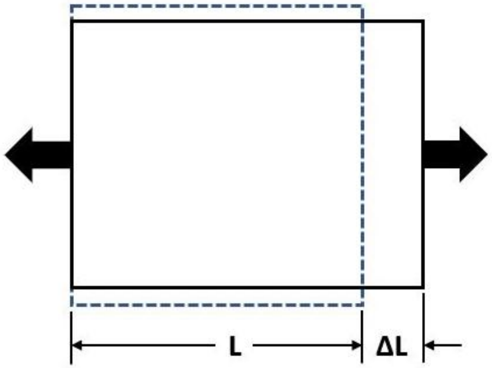
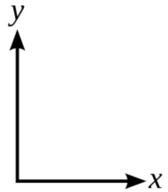


Fig 1a. Normal strain, ε

ε = ΔL / L (mm/mm)

Poisson's ratio

$$\nu = \frac{-\varepsilon_y}{\varepsilon_x}$$



Shear Stress

- Shear stress is the stress acting parallel to the stress element.
- Changes only the shape of the stress elements
- Volume is constant.

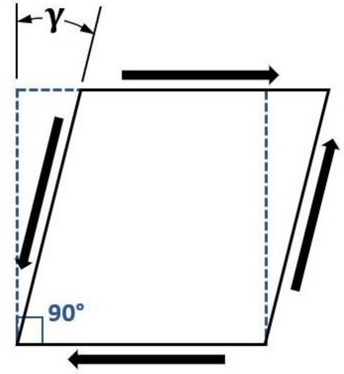


Fig 1b. Shear strain, γ

γ = Change in 90° angle (radians)

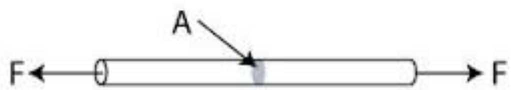
Types of Stresses

Stress

- **Normal Stress**
  - Axial Stress
  - Bending Stress
- **Shear Stress**
  - Direct (Pure) Shear Stress
  - Torsional Shear Stress
  - Transverse(Flexural) Shear Stress

Axial Stress

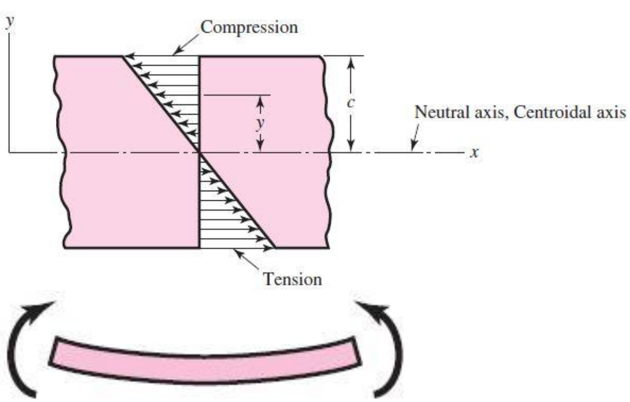
- **Normal stress** due to axial load.
- Considered to be uniformly distributed on the cross section of an element.



$$\sigma = \frac{F}{A}$$

Bending Stress

- **Normal stress** due to bending moment.
- Maximum & minimum at the surface and zero at the neutral axis.
- Neutral axis is the axis coincides with the axis of symmetry of the parts.



$$\sigma = \frac{My}{I}$$

$$\sigma_{max} = \frac{Mc}{I}$$

M = bending moment
y = distance of the element from neutral axis.
I = area moment of inertia
(ability of beam to resist against bending)

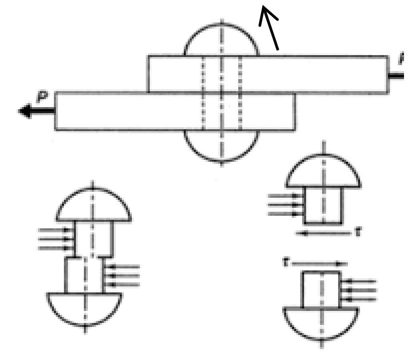
For circular solid beam

$$I = \frac{\pi d^4}{64} \qquad \sigma_b = \frac{32M}{\pi d^3}$$

Direct (Pure) Shear Stress

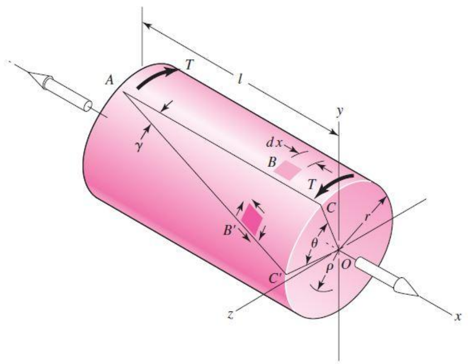
- Occurs when there is a shearing action with no bending.
- Bolts, rivets, pins etc.

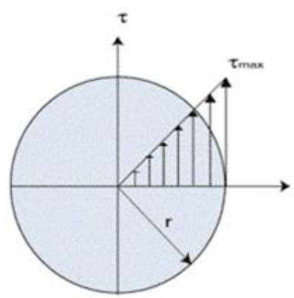
$$\tau = \frac{V}{A}$$



Torsional Shear Stress

- The internal resistance to twisting is the torsional shear stress.
- Some power transmission elements **(gears, pulleys, chains etc.)** creates torque on the shaft and the shaft will twist as a result.
- Zero at the neutral axis and maximum at the surface of the element





T = torque
ρ = distance of the element from neutral axis.
J = polar moment of inertia
(ability of beam to resist torsion)

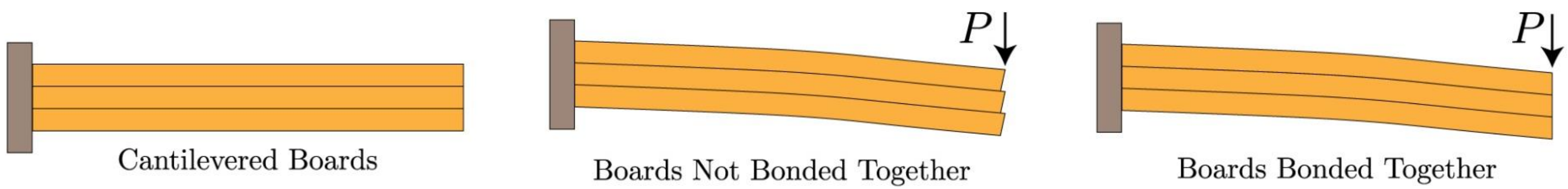
$$\tau = \frac{T\rho}{J} \qquad \tau_{max} = \frac{Tr}{J}$$

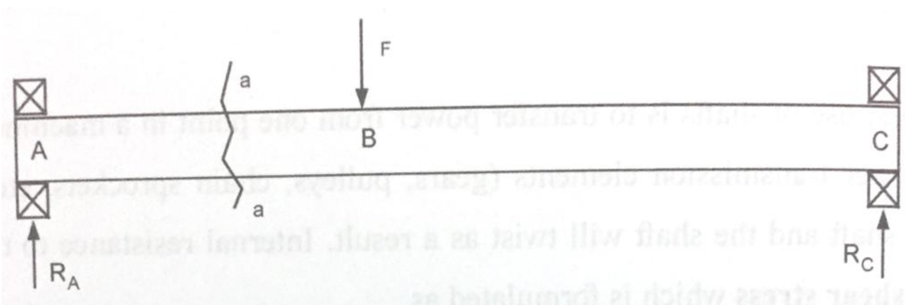
For circular solid beam

$$J = \frac{\pi d^4}{32} \qquad \tau = \frac{16T}{\pi d^3}$$

Transverse (Flexural) Shear Stress

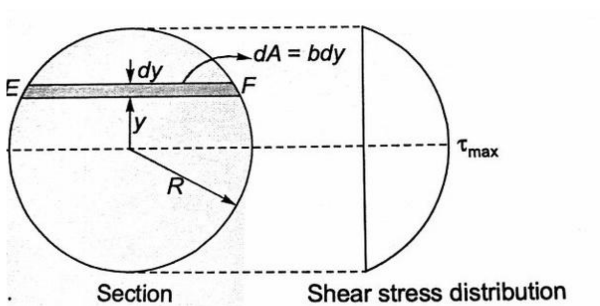
- In addition to normal stresses induced by bending; **transverse shear stresses** are induced between the elements providing that bending moment varies along the length of the beam.





$$\tau = \frac{QV}{Ib_y}$$

V= shear force
Q = first moment of the area.
I= area moment of inertia
b_y= width of the section at the particular distance from the neutral axis.

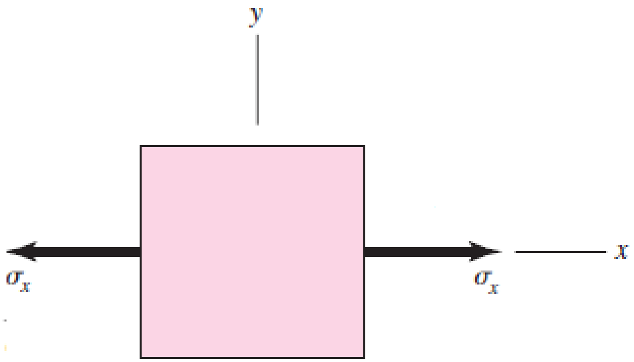


$\tau_{max} = 4V / 3A$ *for solid circular beam*

$\tau_{max} = 2V / A$ *for hollow circular section*

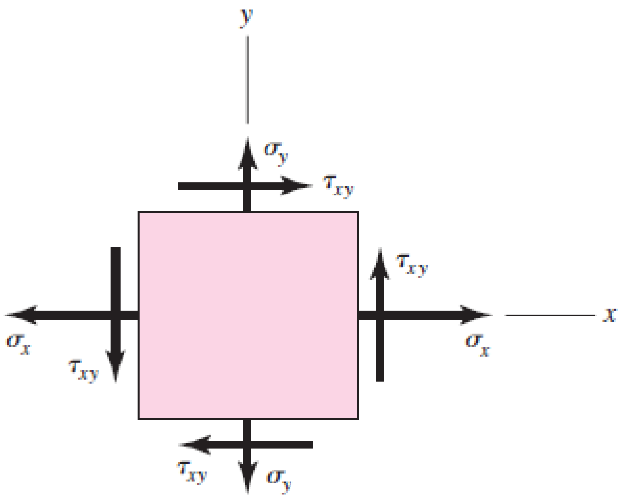
$\tau_{max} = 3V / 2A$ *for rectangular section*

- Loading conditions may result different stress states on the element within the body of a mechanical element.



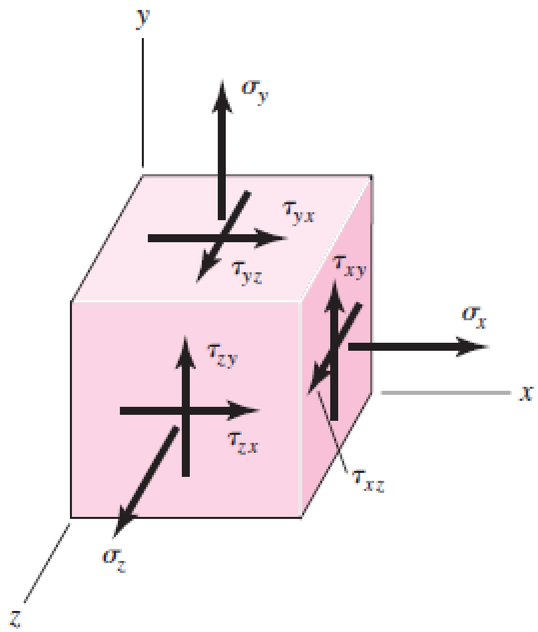
<u>Uniaxial stress state</u>

(a bar loaded in ***simple tension or compression****)*



<u>Biaxial stress state</u>

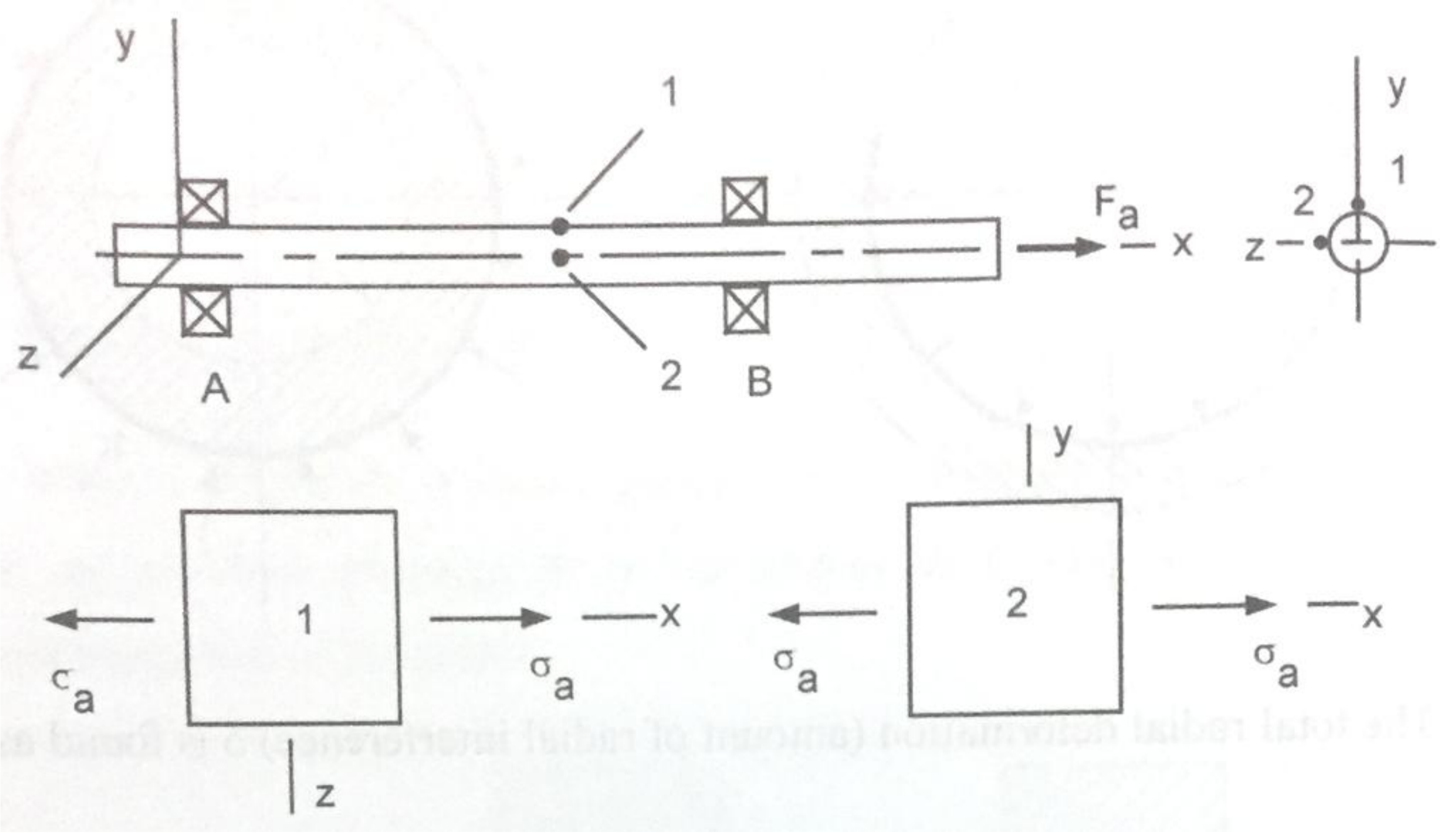
(a shaft loaded by ***bending and torsion****)*



<u>Triaxial stress state</u>

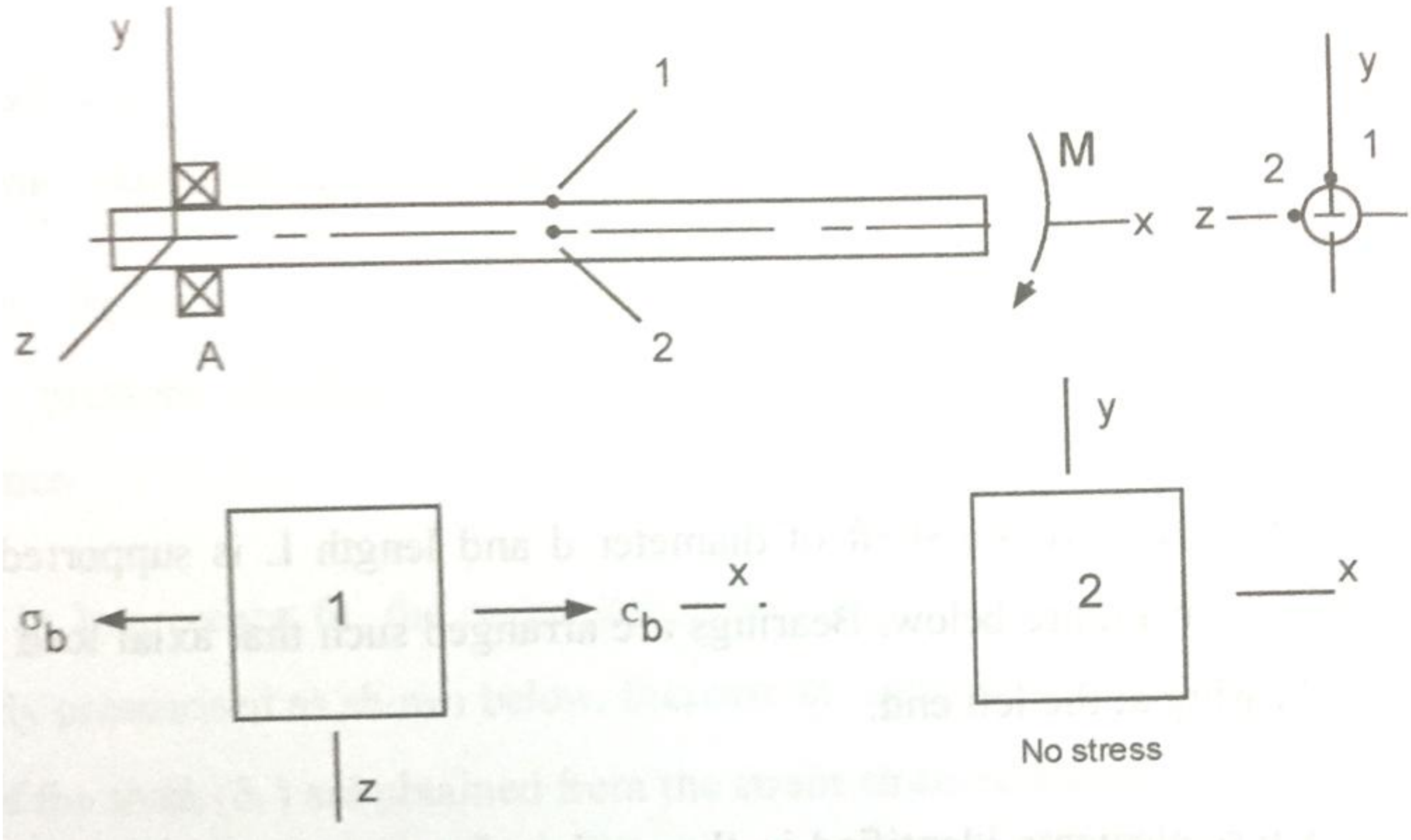
*(****pressure cylinder*** *with closed ends or* ***Hertz contact stresses*** *on gears)*

- **Axial load is acting overhang** *Axial load is carried by left hand support (bearing at A)*

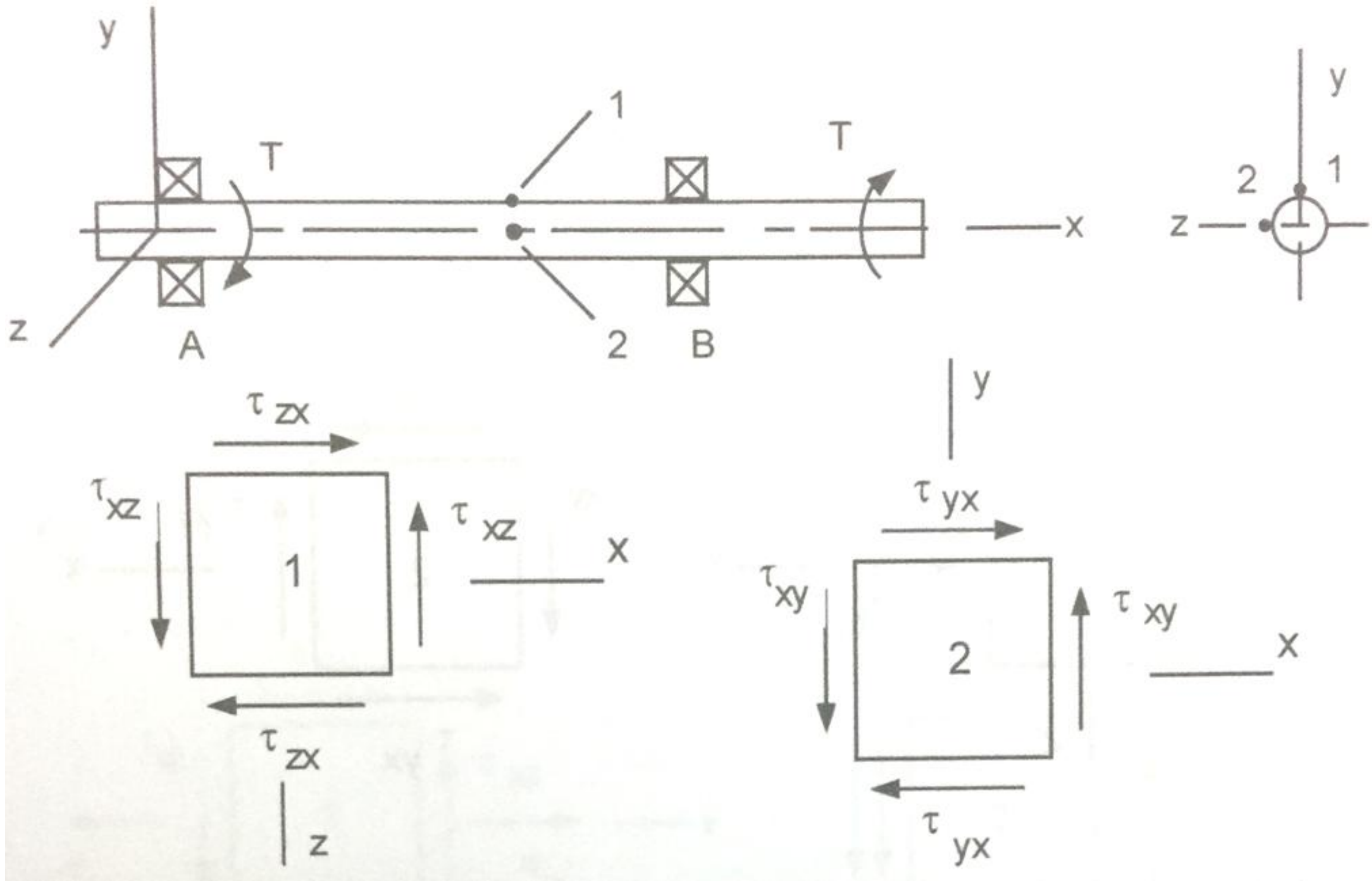


******If the right bearing (bearing at B) were to carry the axial (thrust) load*, *then there will be* ***no stress on the elements*** *between the bearings.*

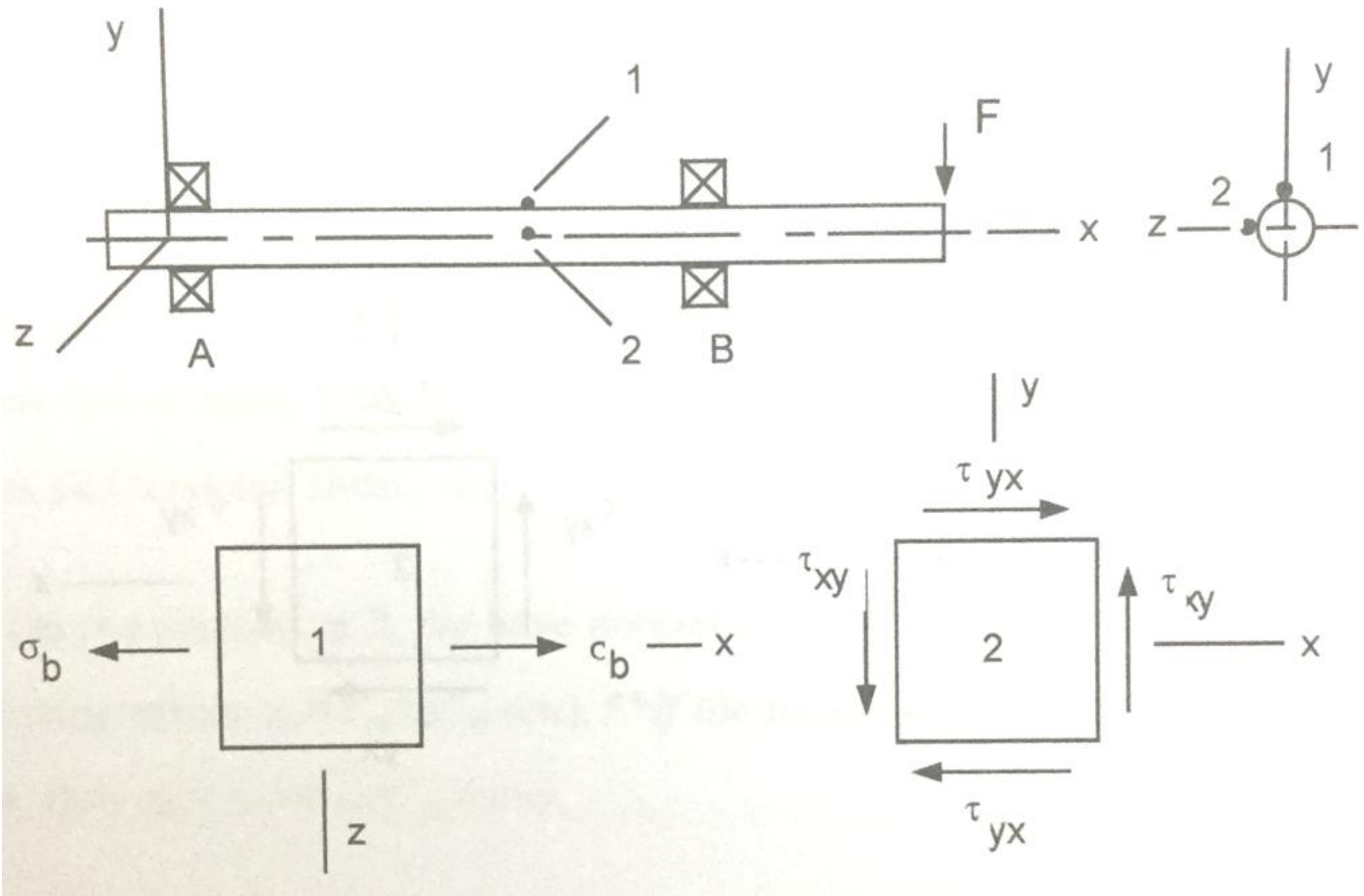
- **Bending moment is acting only**



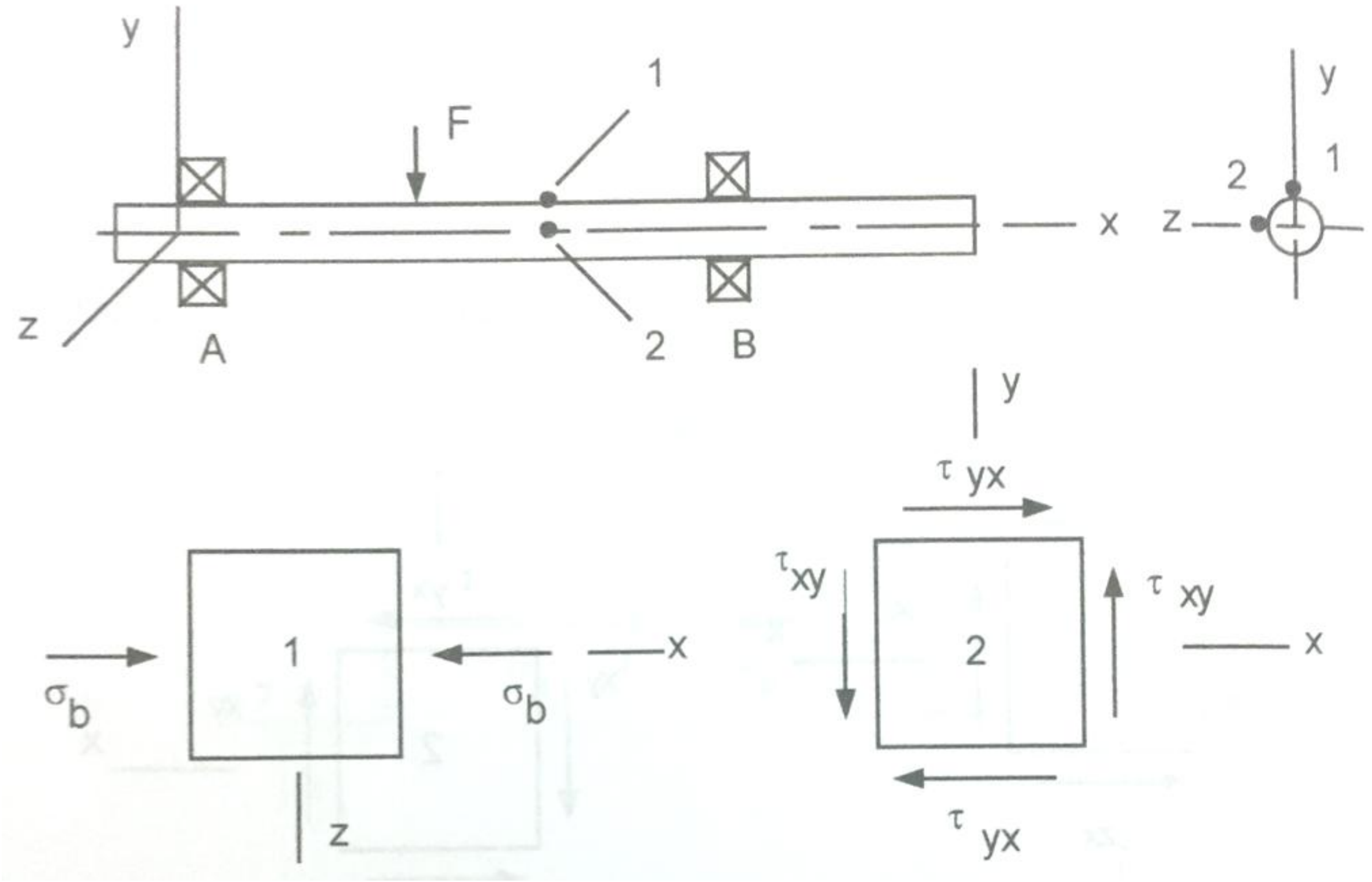
➤ **Torsional load is acting only**



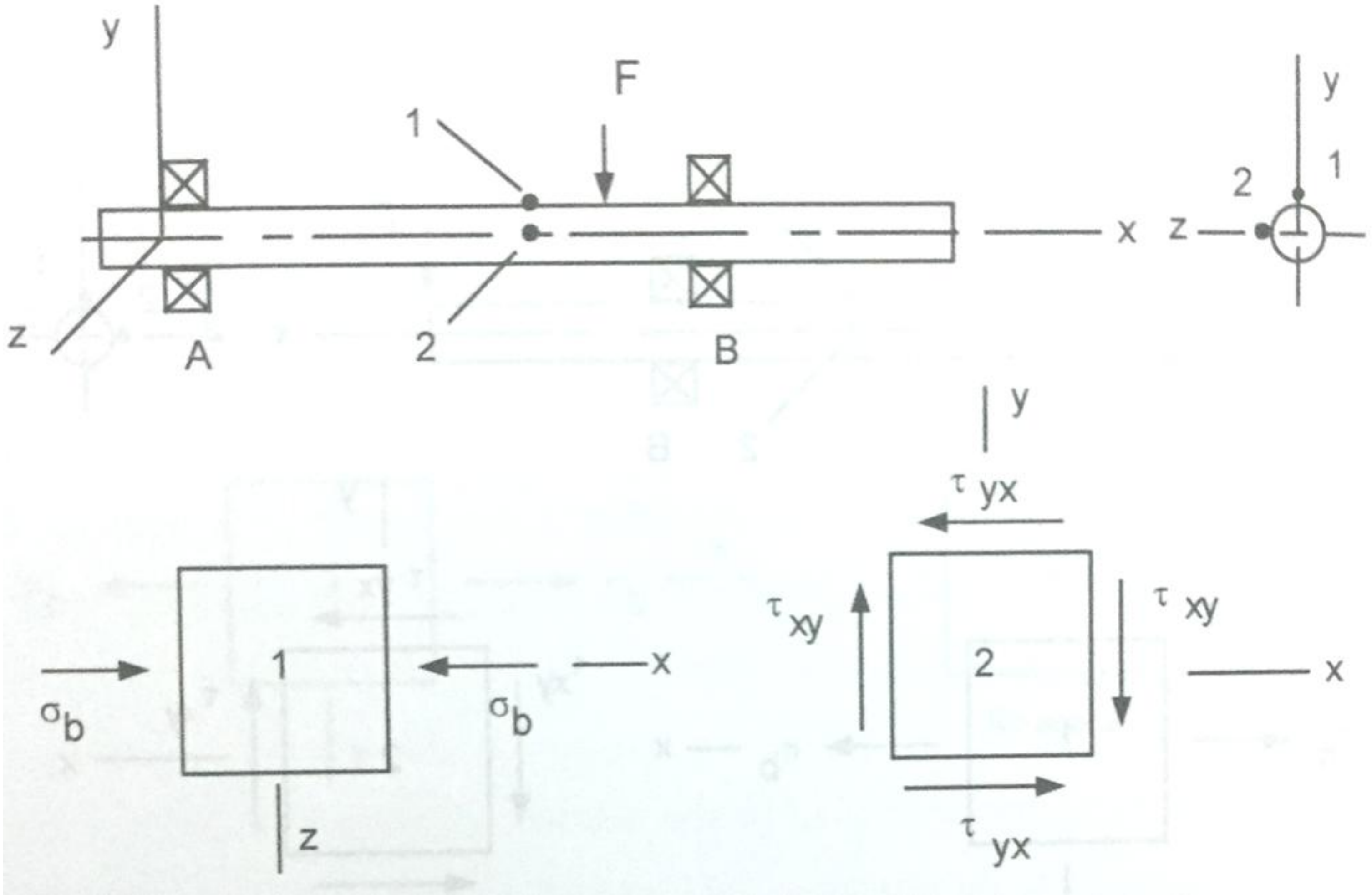
- **Bending load is acting only** ***(F is acting on overhang)***



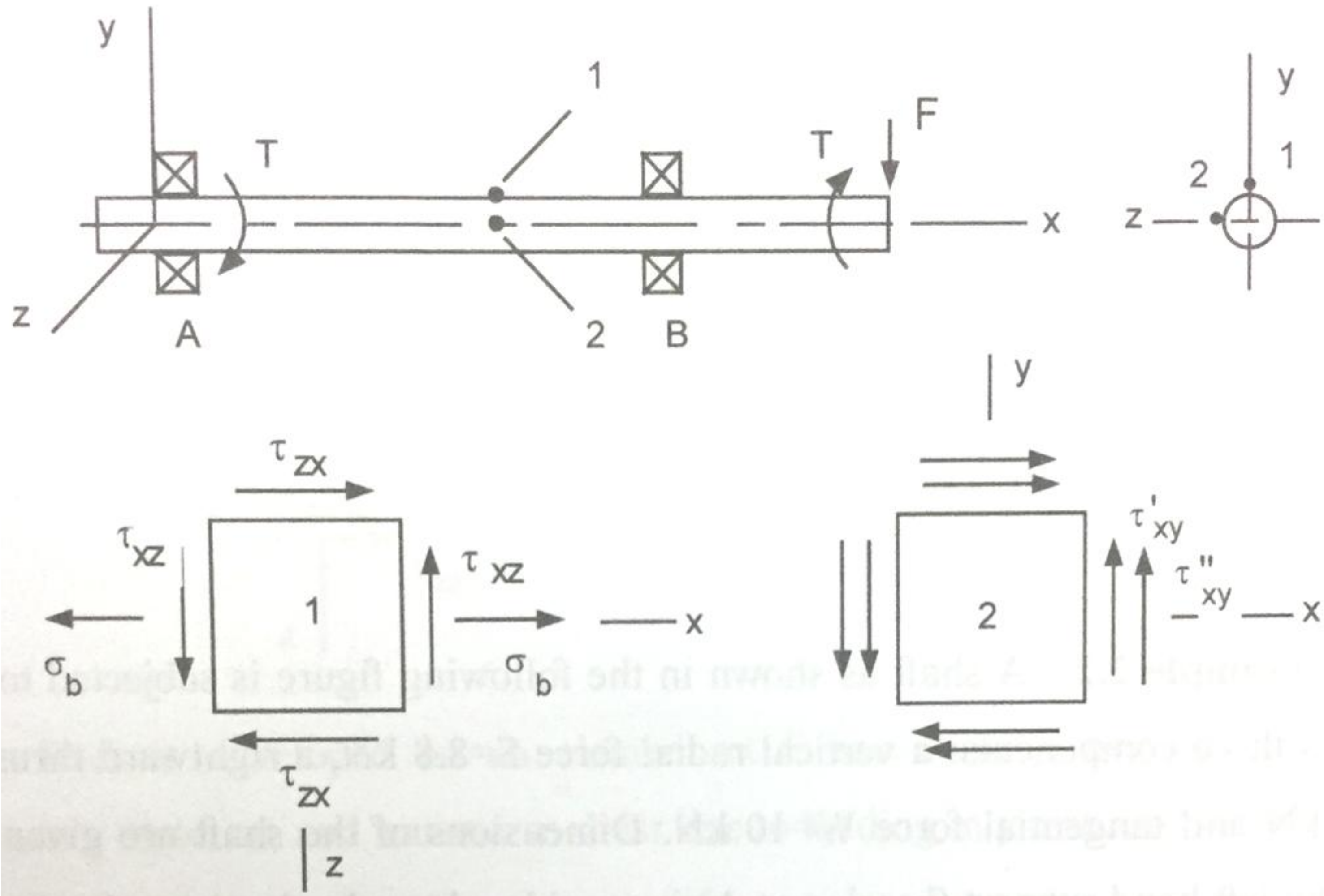
- **<u>Bending load is acting only</u>** ***(F is acting between bearings and to the left of stress elements)***



- **<u>Bending load is acting only</u>** ***(F is acting between bearings and to the right of stress elements)***

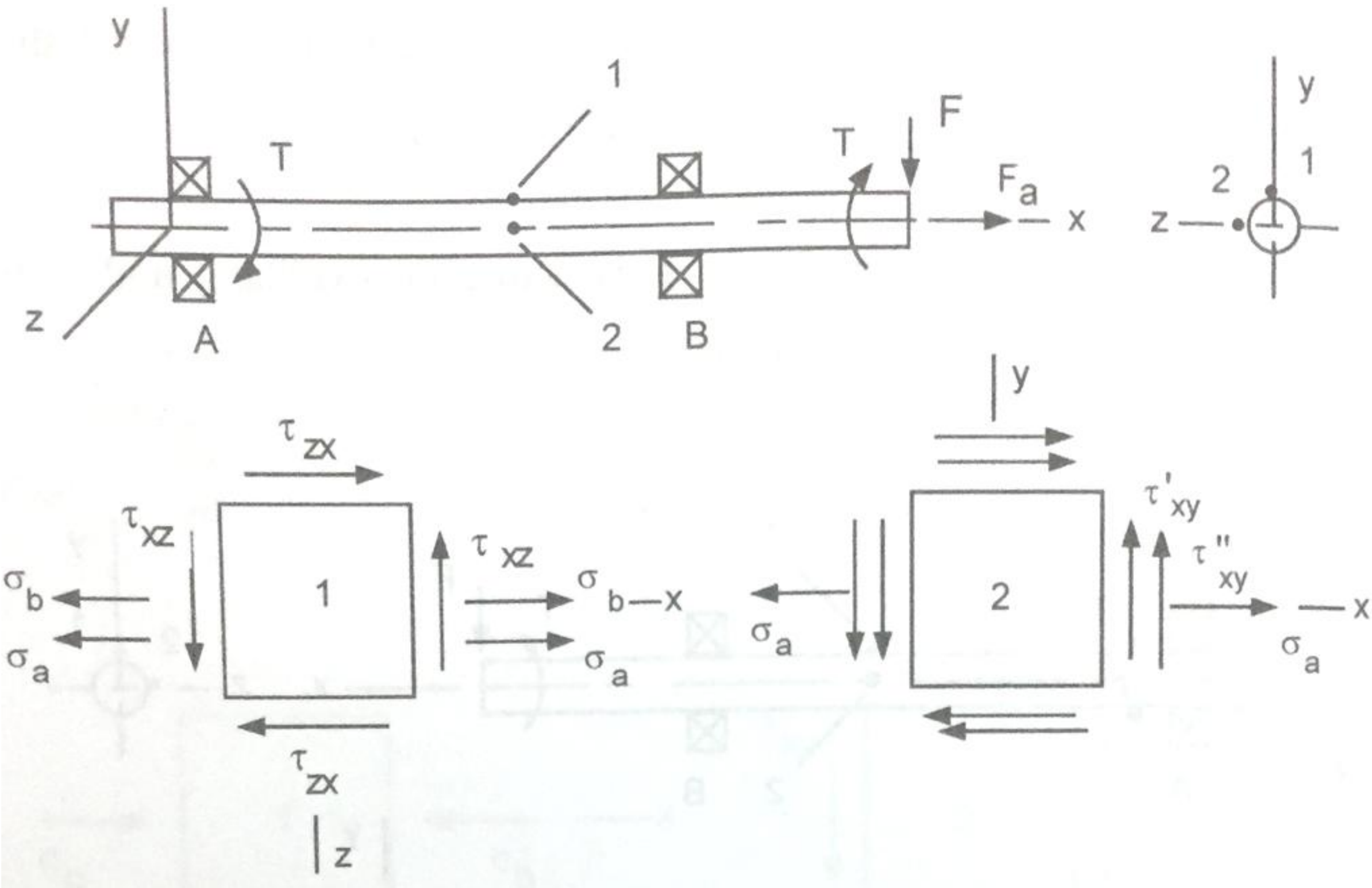


Bending plus torsional loads



Bending plus torsional plus axial loads

Axial load is carried by left hand support (bearing at A)

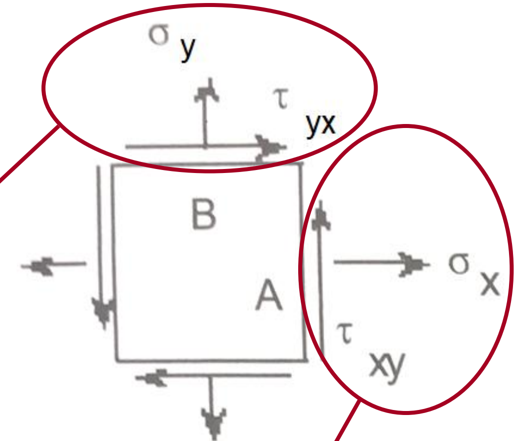


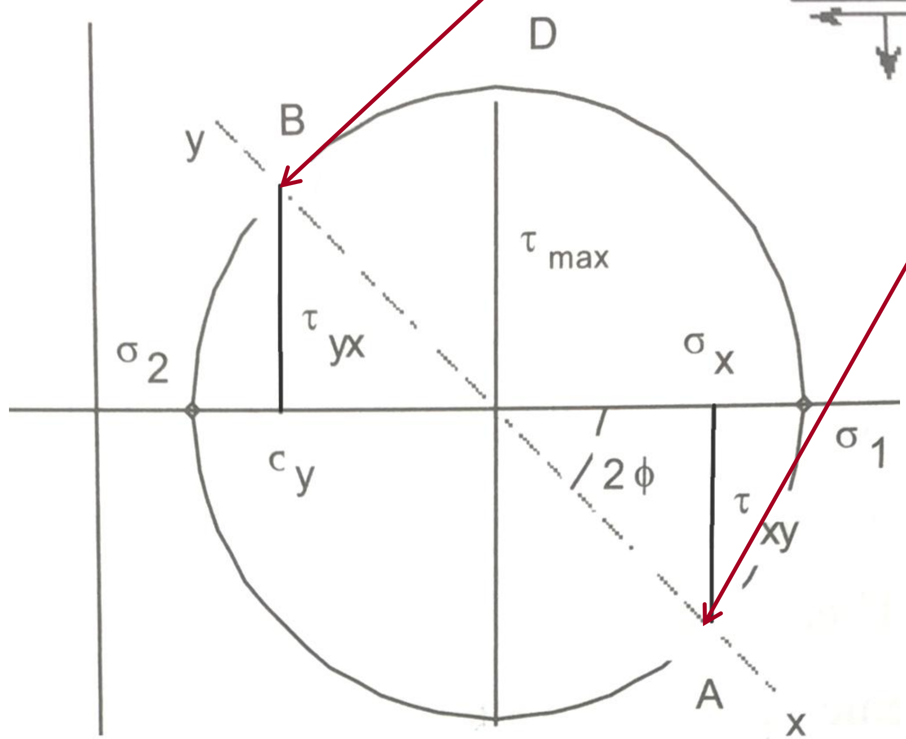
- Whatever the stress state is, designers' job is to determine whether the machine element will fail or not *under given loading condition*.
- Materials exhibit only **a unique value of the strength** *(yield strength "S_y", ultimate strength "S_u")*.
- Therefore, there must be some means to combine stress components in such a way that the resulting stresses on a machine component will be compared with the strength of the material **to determine safeness of the design**.
- Mohr's circle is the special mean for the determination of these parameters.

- **By using Mohr’s circle**, it is possible to determine maximum normal stresses and **the orientation of the stress element** such that there exist only these maximum normal stresses acting on the faces.
- That special orientation is named as **principal direction** and the stresses are called as **principal stresses**.
- There is another orientation of the stress element such that shear stresses reach to extreme values.
- **Principal stresses** are used in the determination of the safeness of the elements according to different failure criteria *(will be discussed in next chapters)*

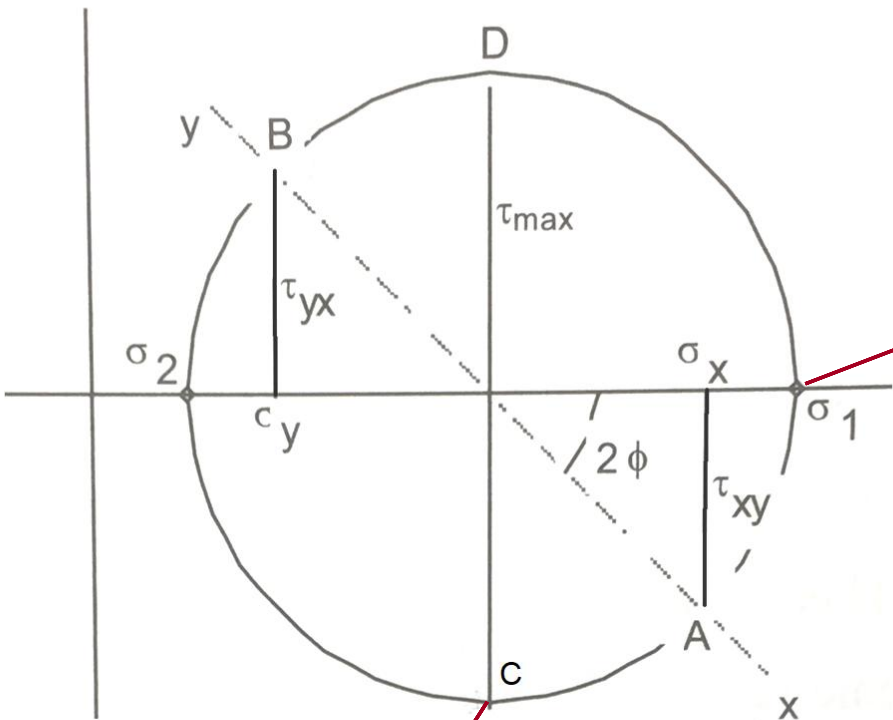
- **A two dimensional Mohr's circle** can be constructed if we know the normal stresses σ_x , σ_y and τ_{xy} .
- The sign conventions should be used.
  - **Tensile stresses (positive)** are **to the right**.
  - **Compressive stresses (negative)** are **to the left**.
  - **Clockwise shear stresses** are plotted **upward**.
  - **Counterclockwise shear stresses** are plotted **downward**.

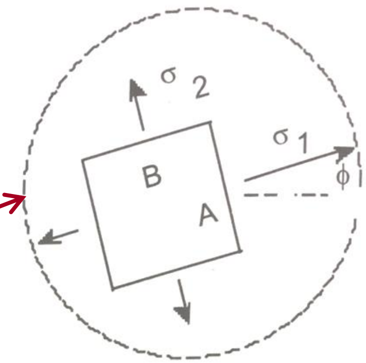
Stress element





Stresses acting on faces A and B are represented as ***point A and B*** *in the Mohr's circle, respectively.*





Principal stresses

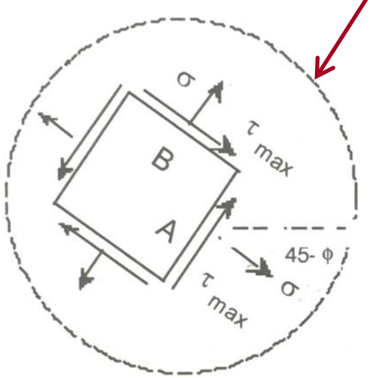
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

Principal direction

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum shear stresses

$$\tau_{1,2} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$



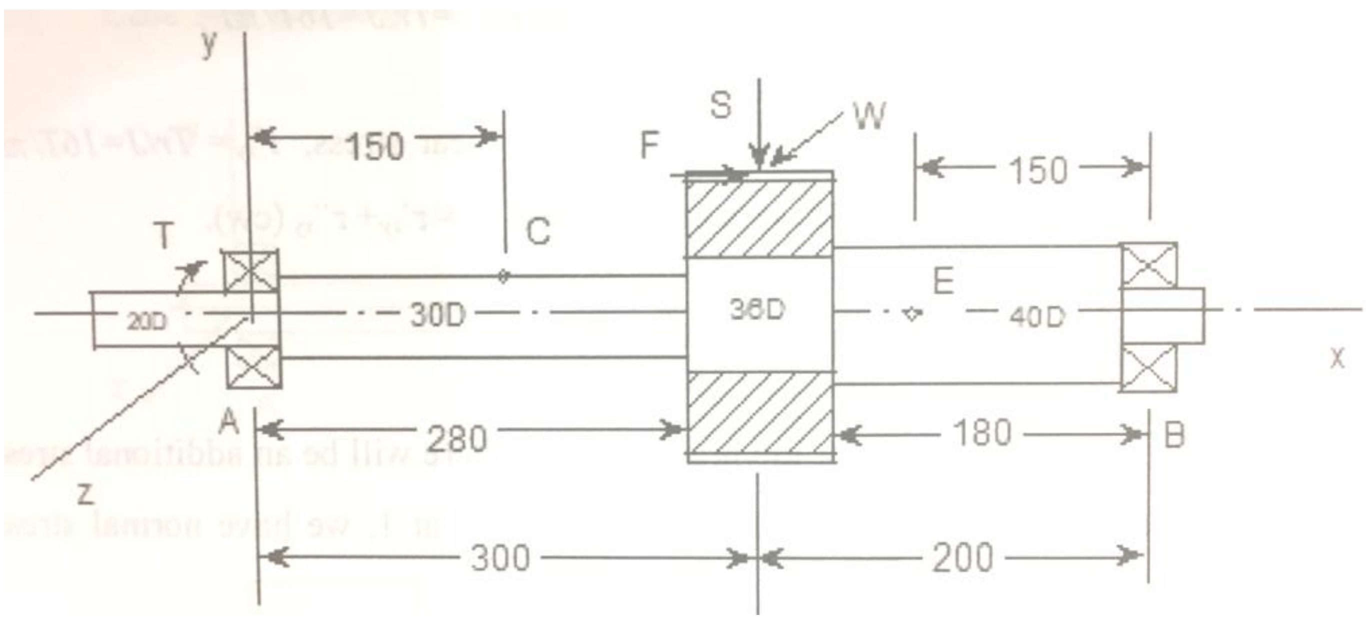
Principal direction for max. shear stress

$$\tan 2\phi' = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

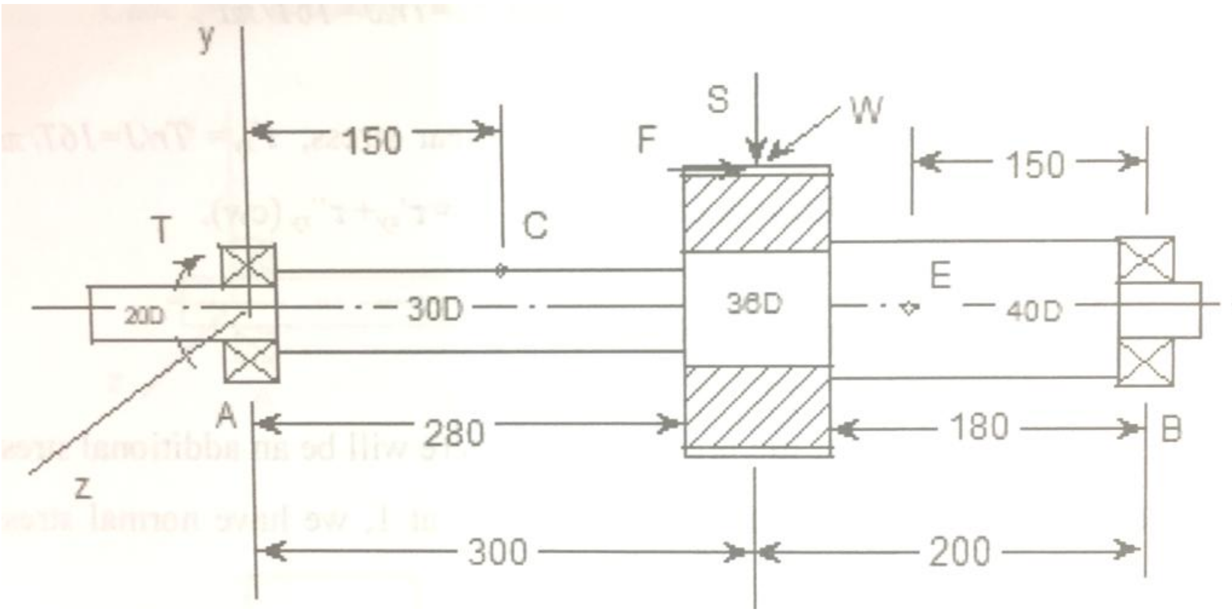
A shaft as shown in the Figure is subjected to a load which has three components; a vertical radial force S=8.8 kN, a rightward thrust force of F=15 kN and tangential force W=10 kN. Dimensions of the shaft are given in the figure. The left hand support (bearing at A) is considered to take thrust load. Two stress elements located on the surface and identified in the sections shown are: C on top; E is on the front side.

a) Draw each of the stress elements properly oriented with respect to xyz, show the stresses which act upon them.

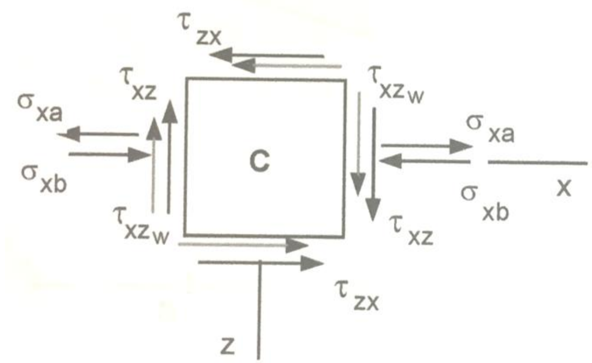
b) Calculate the principle stresses at point E. Show the proper orientation (principal direction) of the element.

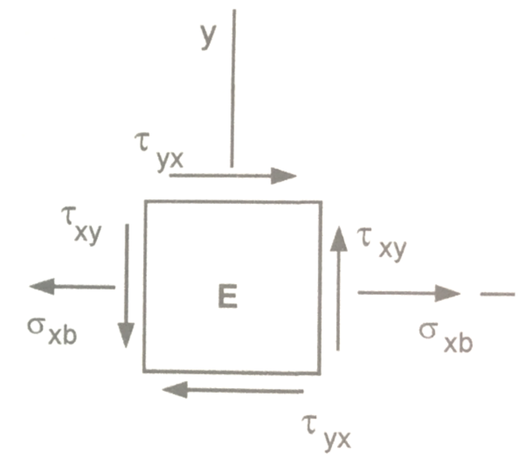


Solution of Example 2.2

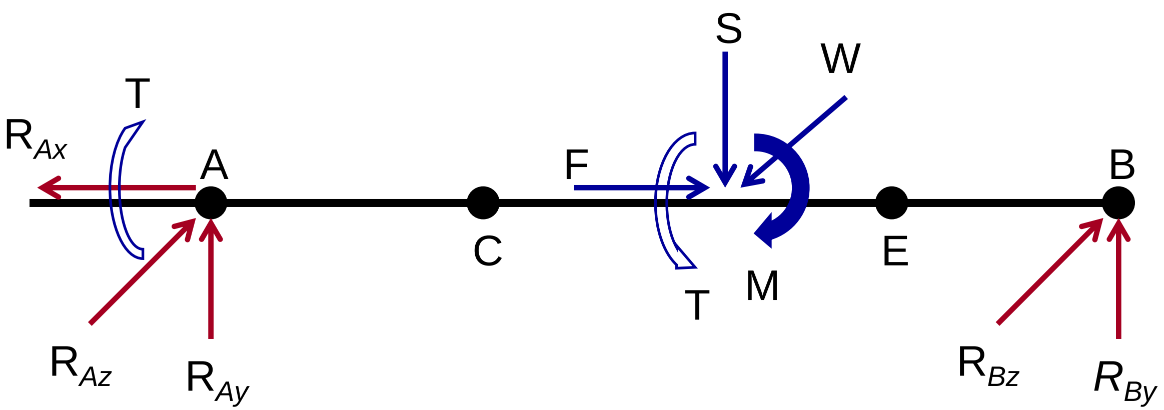


Stress Elements

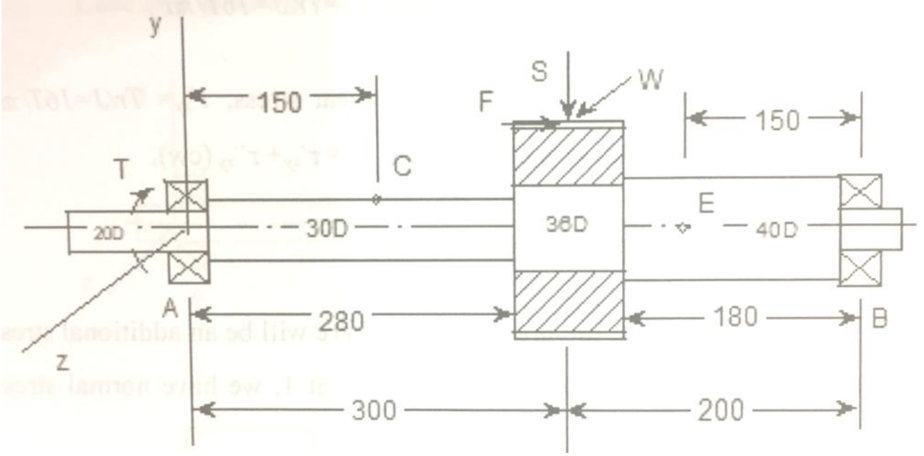


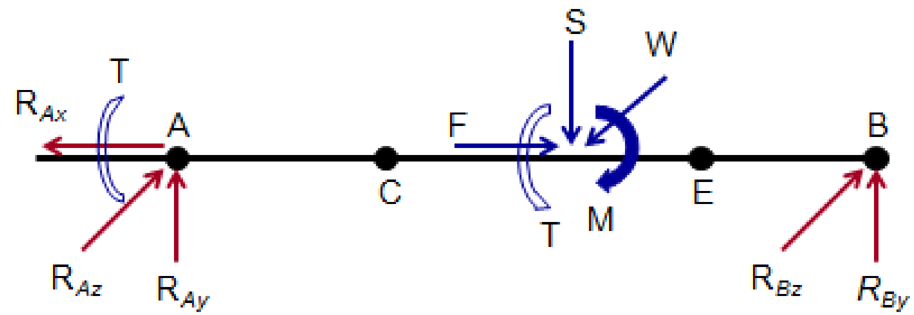


Free Body Diagram



Solution of Example 2.2





Reaction in x-direction

$$R_{Ax} = F = 15kN$$

Reaction in y-direction

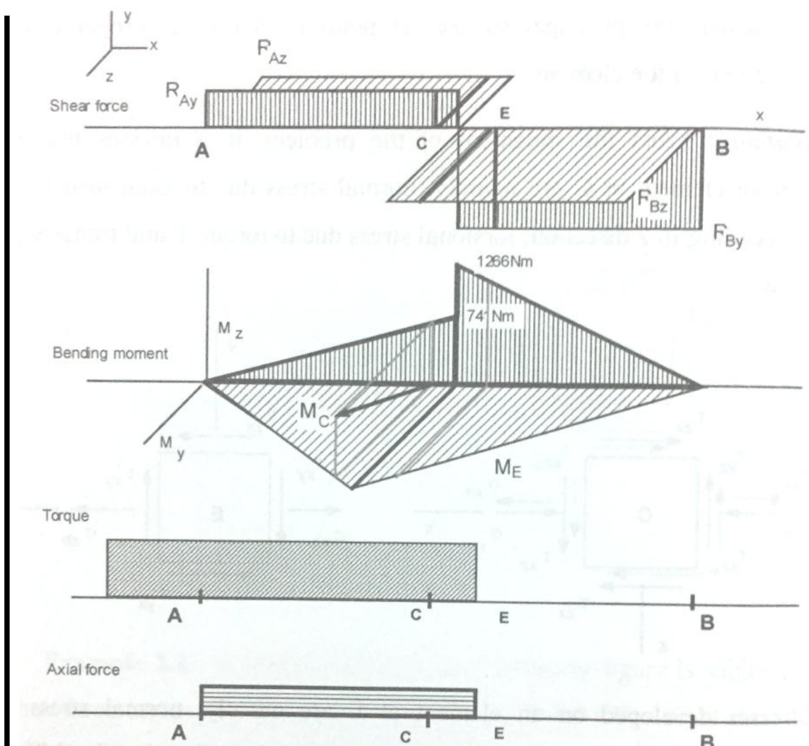
$$R_{Ay} = \frac{S(200) - F(35)}{500} = \frac{8.8(200) - 15(35)}{500} = 2.47kN$$

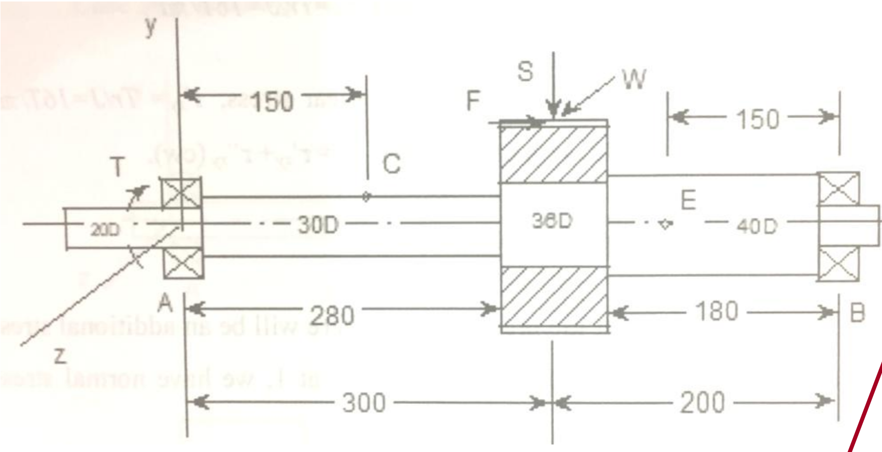
$$R_{By} = S - R_{Ay} = 8.8 - 2.47 = 6.33kN$$

Reaction in z-direction

$$R_{Az} = \frac{W(200)}{500} = \frac{10(200)}{500} = 4kN$$

$$R_{Bz} = W - R_{Az} = 10 - 4 = 6kN$$



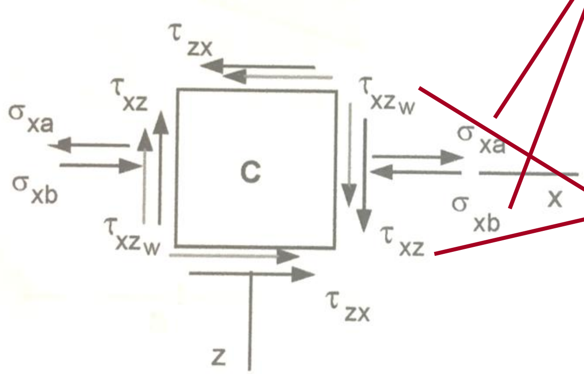


Bending moment at point C

$$M_C = R_{Ay}\left|AC\right| = 2.47(150) = 370.5\,Nm$$

Torque at point C

$$T_C = W(35) = 10(35) = 350\,Nm$$



At point C, we have normal stresses due to bending moment and axial load F

$$\sigma_{xb} = -\frac{32M_C}{\pi d^3} = \frac{32(370.5)(10)^3}{\pi(30)^3} = -139.77\,MPa$$

$$\sigma_{xa} = \frac{F}{A} = \frac{4F}{\pi d^2} = \frac{4(15000)}{\pi(30)^2} = 21.22\,MPa$$

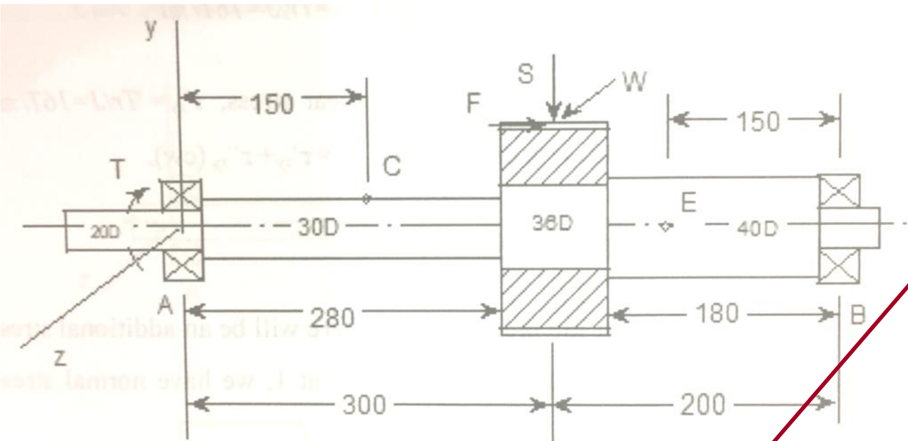
The resultant normal stress

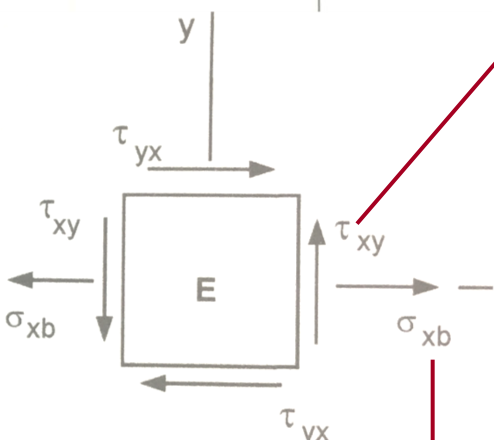
$$\sigma_x = \sigma_{xb} + \sigma_{xa} = -139.77 + 21.22 = -118.55\,MPa$$

There are two shear stress components at point C

$$\tau_{xz} = -\frac{16T_C}{\pi d^3} = \frac{16(350)(1000)}{\pi(30)^3} = 66.02\,MPa$$

$$\tau_{xzw} = \frac{4V}{3A} = \frac{4R_{Az}}{3A} = \frac{4(4)(4000)}{3\pi(30)^2} = 7.54\,MPa$$





Bending moment at point E

$$M_E = R_{Bz}\left|BE\right| = 6(150) = 900\,Nm$$

Normal stress due to bending

$$\sigma_{xb} = -\frac{32M_E}{\pi d^3} = \frac{32(900)(10)^3}{\pi(40)^3} = 143.24MPa$$

Transverse shear stress due to force

$$\tau_{xy} = \frac{4V}{3A} = \frac{4(6330)}{3\pi(20)^2} = 6.72MPa$$

Principal stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

$$\sigma_{1,2} = \frac{143.24}{2} \pm \sqrt{(71.62)^2 + (6.72)^2} = 71.62 \pm 71.93$$

$$\sigma_1 = 143.55MPa \qquad \sigma_2 = -0.31MPa$$

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x} = \frac{2(6.72)}{143.24} = 0.094$$

$$\phi = 2.68^\circ\ (CCW)$$

ME 307 – Machine Elements I

Chapter 2

Stress Analysis (Part II)

Mechanical Engineering
University of Gaziantep

Dr. Sadık Olguner
Dr. Hakan Çandar

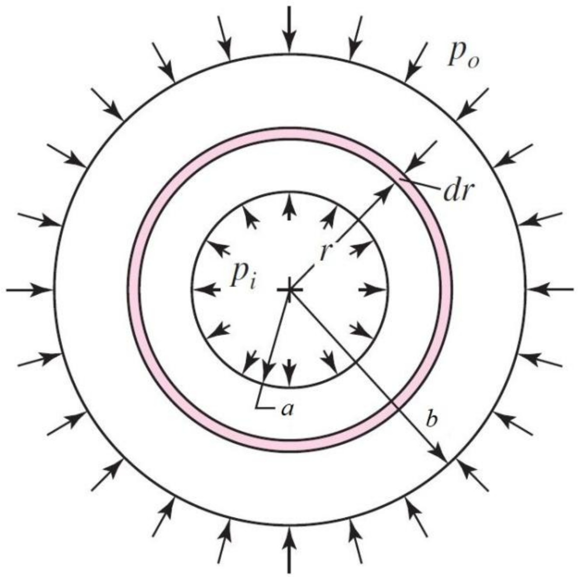
- **Pressurised cylinder** is a cylinder that pressurised internally and externally.
- Pressurised cylinders are used in many industrial application such as *pressure vessels, hydraulic cylinders, gun barrels and pipes carrying fluids at high pressure*.
- **By writing force equilibrium in vertical direction** and assuming that elongation in longitudinal direction remains constant *(plane stress case)*, tangential (hoop) stress and radial stress can be calculated by **Lame's equations**.

$$\sigma_t = \frac{p_i a^2 - p_o b^2 - a^2 b^2 (p_o - p_i)/r^2}{b^2 - a^2}$$

$$\sigma_r = \frac{p_i a^2 - p_o b^2 + a^2 b^2 (p_o - p_i)/r^2}{b^2 - a^2}$$

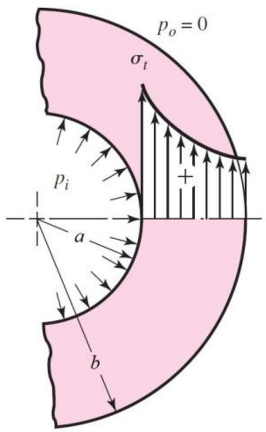
<u>If ends are closed</u>

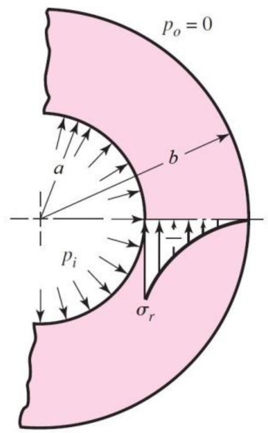
$$\sigma_l = \frac{p_i a^2}{b^2 - a^2}$$



Pressurised Cylinders

If the external pressure is zero (p_o=0)





$$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

$$\sigma_r = \frac{a^2 p_i}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

At the inner surface

$$r = a$$

$$\sigma_r = -p_i$$

$$\sigma_t = p_i \frac{b^2 + a^2}{b^2 - a^2}$$

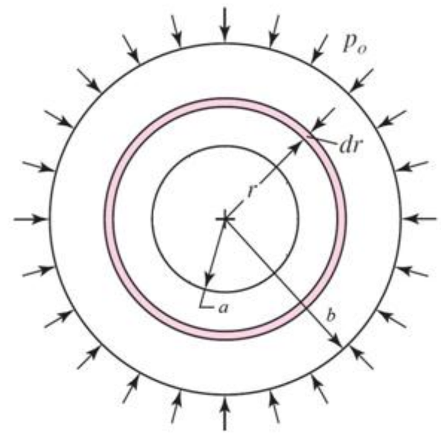
At the outer surface

$$r = b$$

$$\sigma_r = 0$$

$$\sigma_t = p_i \frac{2a^2}{b^2 - a^2}$$

If the internal pressure is zero (p_i=0)



$$\sigma_t = -p_o \frac{b^2}{b^2 - a^2}\left(1 + \frac{a^2}{r^2}\right)$$

$$\sigma_r = -p_o \frac{b^2}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right)$$

At the inner surface

$$r = a$$

$$\sigma_r = 0$$

$$\sigma_t = -p_o \frac{2b^2}{b^2 - a^2}$$

At the outer surface

$$r = b$$

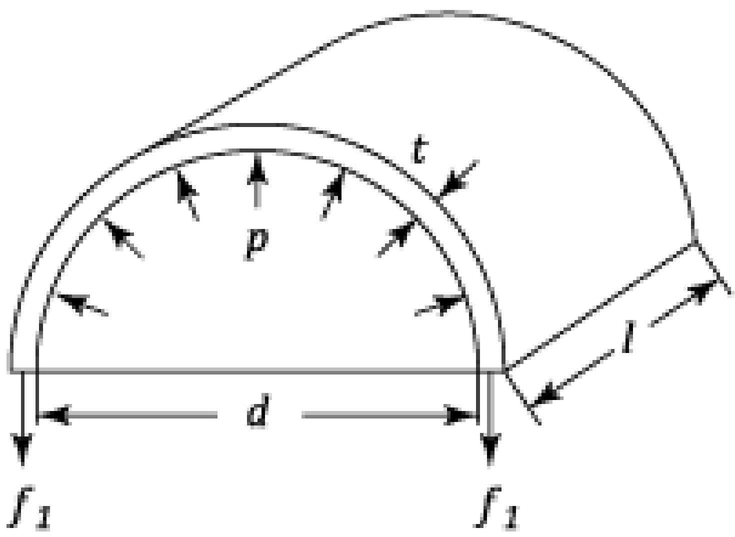
$$\sigma_r = -p_o$$

$$\sigma_t = -p_o \frac{b^2 + a^2}{b^2 - a^2}$$

- When the wall thickness of a cylindrical pressure vessel is about one-twentieth, or less, of its radius, **the radial stress** that results from pressurizing the vessel **is quite small compared with the tangential stress**.
- The internal force is resisted by tangential stress acting uniformly over the stressed area.

Internally pressurised thin walled cylinder

($t/r \le 20$)

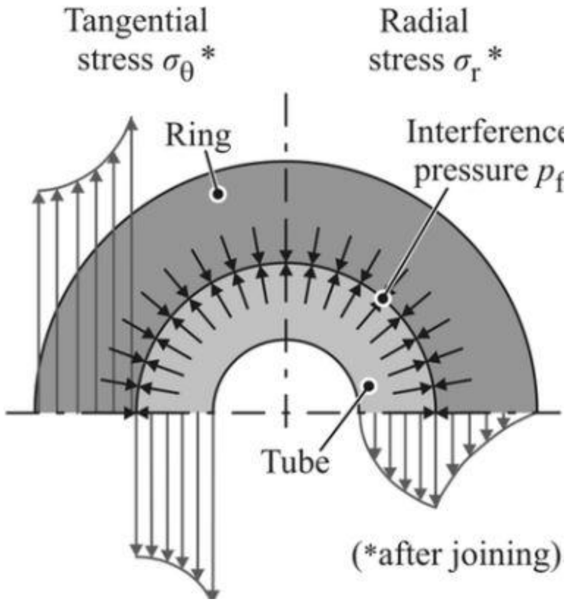


$$\sigma_t = \frac{pd_i}{2t}$$

$$\sigma_r = -p$$

$$\sigma_l = \frac{pd_i}{4t}$$

- **Interference fit** occurs when a shaft with a larger diameter is to be fitted to a hole with a smaller diameter.
- Interference fit can be obtained by heating and/or cooling the members or by applying a certain amount of force in axial direction.
- Interference fit is one of the common mechanical methods used to join two cylindrical components with a certain amount of interference;
  - to transmit torque *(shaft-gear connection)*
  - to create compressive type of residual stress in thick walled cylinders.
- After fitting, a certain amount of interface pressure develops between the members as illustrated in Figure.
- Interface pressure acts as an external pressure for the inner member and as an internal pressure for the outer member.



- The difference between shaft diameter and hole diameter is called as **diametral interference** and it is equal to sum of deformations experienced by inner and outer parts. ($\delta=\delta_o+\delta_i$)
- Increase in radius of the hole (δ_o) and decrease in radius of the shaft (δ_i) are obtained from **strain-stress relation**.

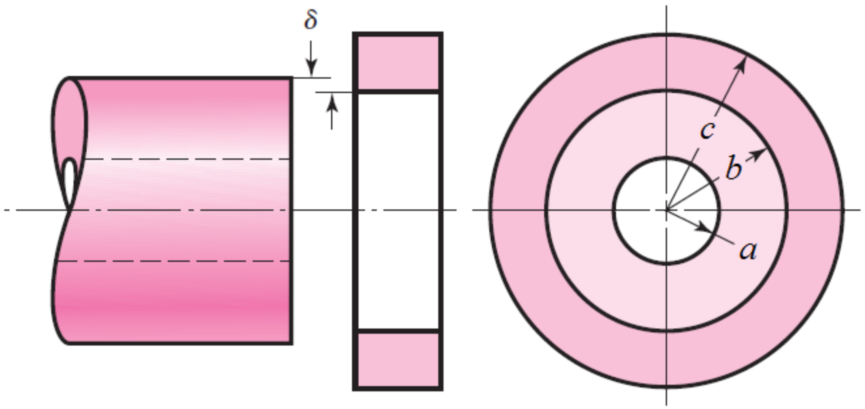
$$\delta_o = \frac{bp}{E_o}\left(\frac{c^2+b^2}{c^2-b^2}+\mu_o\right)$$

$$\delta_i = -\frac{bp}{E_i}\left(\frac{b^2+a^2}{b^2-a^2}-\mu_i\right)$$

$$\delta = \delta_o + \delta_i = \frac{bp}{E_o}\left(\frac{c^2+b^2}{c^2-b^2}+\mu_o\right) + \frac{bp}{E_i}\left(\frac{b^2+a^2}{b^2-a^2}-\mu_i\right)$$

If the materials are the same ($E_o=E_i=E$)

$$p = \frac{E\delta}{b}\left[\frac{\left(c^2-b^2\right)\left(b^2-a^2\right)}{2b^2\left(c^2-a^2\right)}\right]$$



➤ **Lame's equations** for each member of press fitted assembly;

For inner member

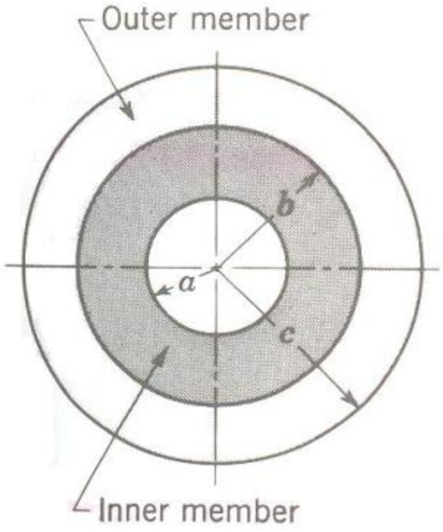
$$\sigma_{ir} = -p\frac{b^2}{b^2-a^2}\left(1-\frac{a^2}{r^2}\right)$$

$$\sigma_{it} = -p\frac{b^2}{b^2-a^2}\left(1+\frac{a^2}{r^2}\right)$$

For outer member

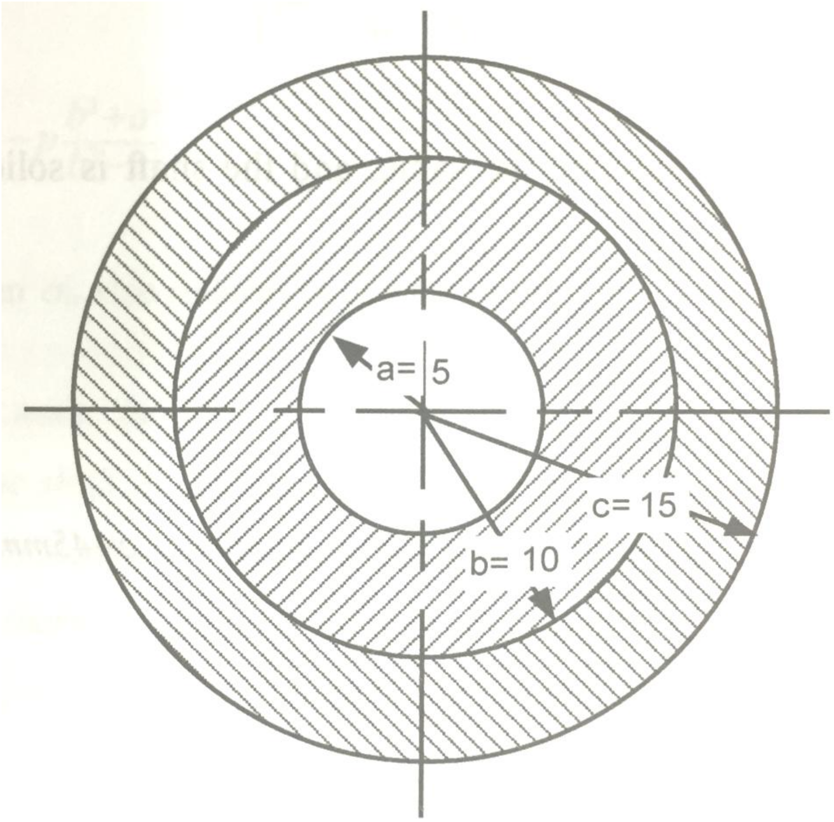
$$\sigma_{or} = p\frac{b^2}{c^2-b^2}\left(1-\frac{c^2}{r^2}\right)$$

$$\sigma_{ot} = p\frac{b^2}{c^2-b^2}\left(1+\frac{c^2}{r^2}\right)$$

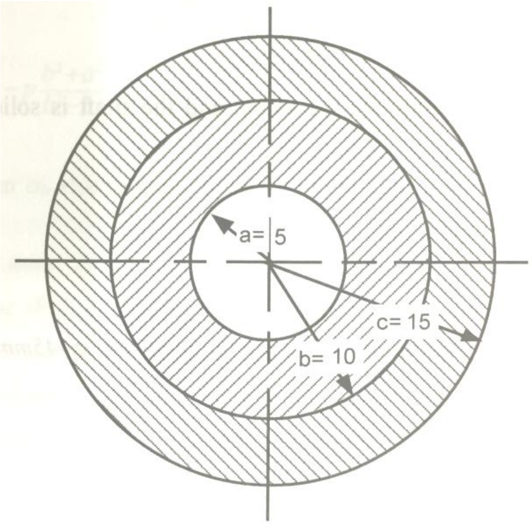


Example 2.10

A gun barrel is assembled by shrinking an outer barrel over an inner barrel so that the maximum principal stress equal to 70% of the yield strength of the material of the barrels. Both barrels are made from steel and have properties of *Sy=530 MPa, E=207 GPa, μ=0.292 (poisson's ratio)*. Inner and outer diameters of the inner barrel are 10 mm and 20 mm, respectively. Outer diameter of the outer barrel is 30 mm. What value of interference should be used in the assembly. Plot the stress distribution on the inner and outer barrel.



Solution of Example 2.10



When two element fitted with an interference fit, contact pressure is

$$\sigma_r = -p$$

Tangential stress on the outer surface of the inner member

$$\sigma_{it} = -p\frac{b^2 + a^2}{b^2 - a^2} = -p\frac{10^2+5^2}{10^2-5^2} = -1.67\,pMPa$$

Tangential stress on the inner surface of the outer member

$$\sigma_{ot} = p\frac{c^2 + b^2}{c^2 - b^2} = p\frac{15^2+10^2}{15^2-10^2} = 2.6\,pMPa$$

Hence, maximum principal stress is σ_{ot}

$$\sigma_{ot} = 0.7S_y = 0.7(530) = 371MPa$$

Interface pressure is

$$p = \frac{\sigma_{ot}}{2.6} = \frac{371}{2.6} = 142.7MPa$$

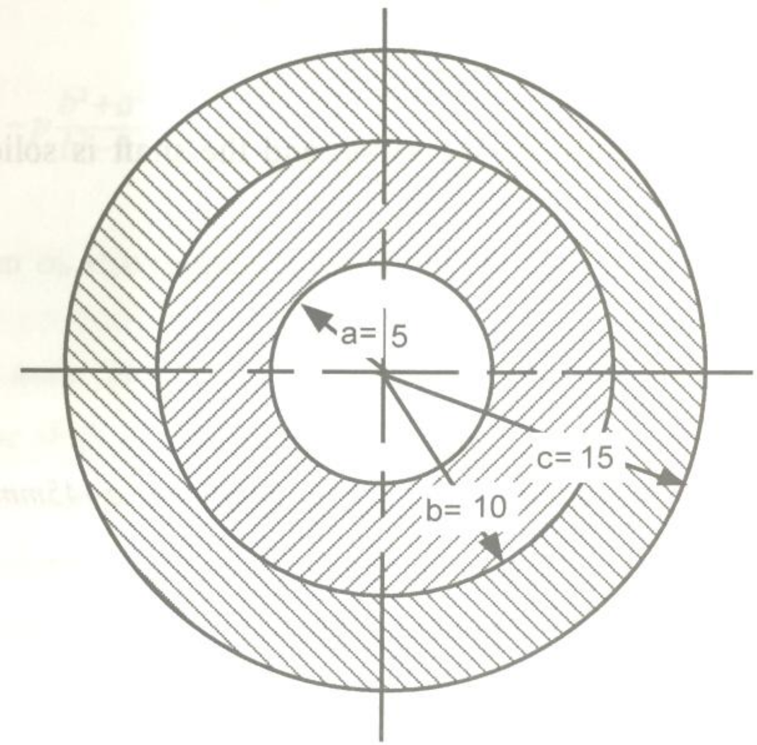
Since both barrels are made from same material

$$\delta = \frac{bp}{E}\left[\frac{2b^2(c^2 - a^2)}{(c^2 - b^2)(b^2 - a^2)}\right]$$

$$\delta = \frac{10(142.7)}{207000}\left[\frac{2(10)^2(15^2 - 5^2)}{(15^2 - 10^2)(10^2 - 5^2)}\right] = 0.029mm$$ *in radiuswise*

$$\delta = 2(0.029) = 0.058mm$$ *in diameterwise*

Solution of Example 2.10



Inner barrel is sujected to external pressure of p=142.7 Mpa. Tangential and radial stresses are;

$$\sigma_{it} = -p\frac{b^2}{b^2 - a^2}\left(1+\frac{a^2}{r^2}\right) = -190.3 + \frac{4576.7}{r^2}$$

$$\sigma_{ir} = -p\frac{b^2}{b^2 - a^2}\left(1-\frac{a^2}{r^2}\right) = -190.3 - \frac{4576.7}{r^2}$$

Outer barrel is sujected to internal pressure of p=142.7 Mpa. Tangential and radial stresses are;

$$\sigma_{ot} = p\frac{b^2}{c^2 - b^2}\left(1+\frac{c^2}{r^2}\right) = 114.2 + \frac{25686}{r^2}$$

$$\sigma_{or} = p\frac{b^2}{c^2 - b^2}\left(1-\frac{c^2}{r^2}\right) = 114.2 - \frac{25686}{r^2}$$

Solution of Example 2.10

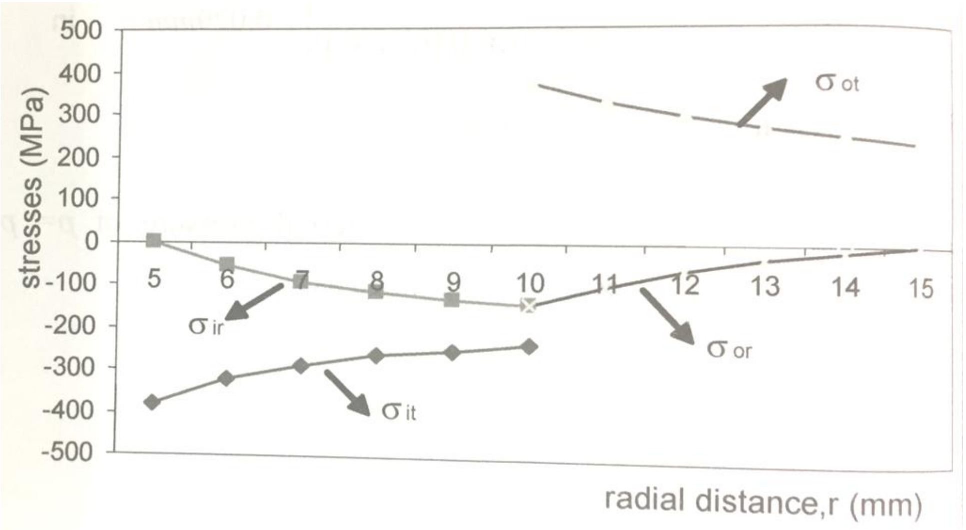
These stresses (σ_{ot} σ_{or} σ_{it} σ_{ir}) are functions of radius r.

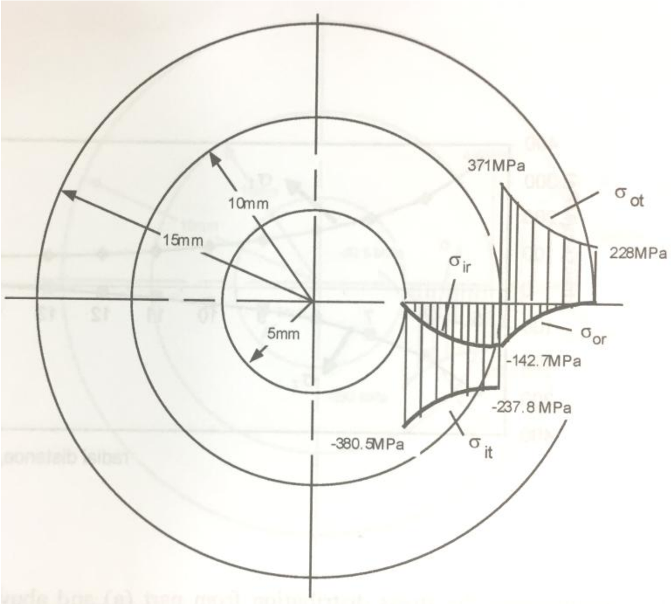
$$\sigma_{it} = -190.3 + \frac{4576.7}{r^2}$$

$$\sigma_{ir} = -190.3 - \frac{4576.7}{r^2}$$

$$\sigma_{ot} = 114.2 + \frac{25686}{r^2}$$

$$\sigma_{or} = 114.2 - \frac{25686}{r^2}$$





On the inner part, *both tangential and radial stresses are of compressive type.* ***At the interface****, maximum radial stress is equal to interface pressure.* ***Tangential stress*** *on the outer member is of tension type.*

ME 307 – Machine Elements I

Chapter 3

Deflection Analysis (Part I)

Mechanical Engineering
University of Gaziantep

Dr. Sadık Olguner
Dr. Hakan Çandar

- **Deflection** is one of the important consideration in the design of machanical elements.
- In some cases, **an element may be strong enough as to carry the loads** without any failure, but **the deflection is so high** that the system may be adversely affected.
- For instance, in the main drive of machine tools, the deflection at the tip of the spindles should not be greater than the tolerable limits. Otherwise, large deflections may create chatter which affects dimensional accuracy and geometrical tolerances *(roundness, cylindricity, concentricity)*.
- As another example, in a power transmission system, the gears are mounted on a solid shaft. If the shaft bends too much, the teeth of the gears can not mesh properly and **the result will be** *noise, wear, and an early failure*.
- In these type of applications, in addition to stress analysis, **deflection at the specific points must be determined** as to finalise the design study.

Deflection under axial loading

When an element is subjected to axial loading, the relationship between deflection and force may be obtained from:

$$\varepsilon = \frac{\delta}{L} = \frac{\sigma}{E} = \frac{F}{AE} \longrightarrow \delta = \frac{FL}{AE} \longrightarrow \frac{F}{\delta} = k = \frac{AE}{L}$$

Resistance of the element against axial deflection

Deflection under torsional loading

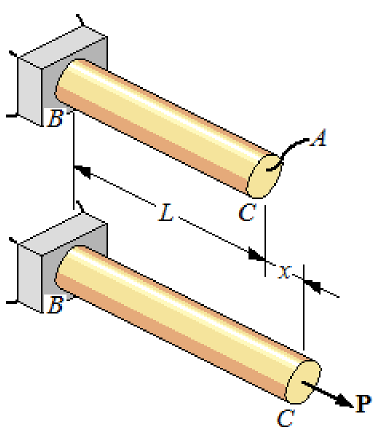
When an element is subjected to torsional loading, the relationship between angular deflection and torque may be obtained from:

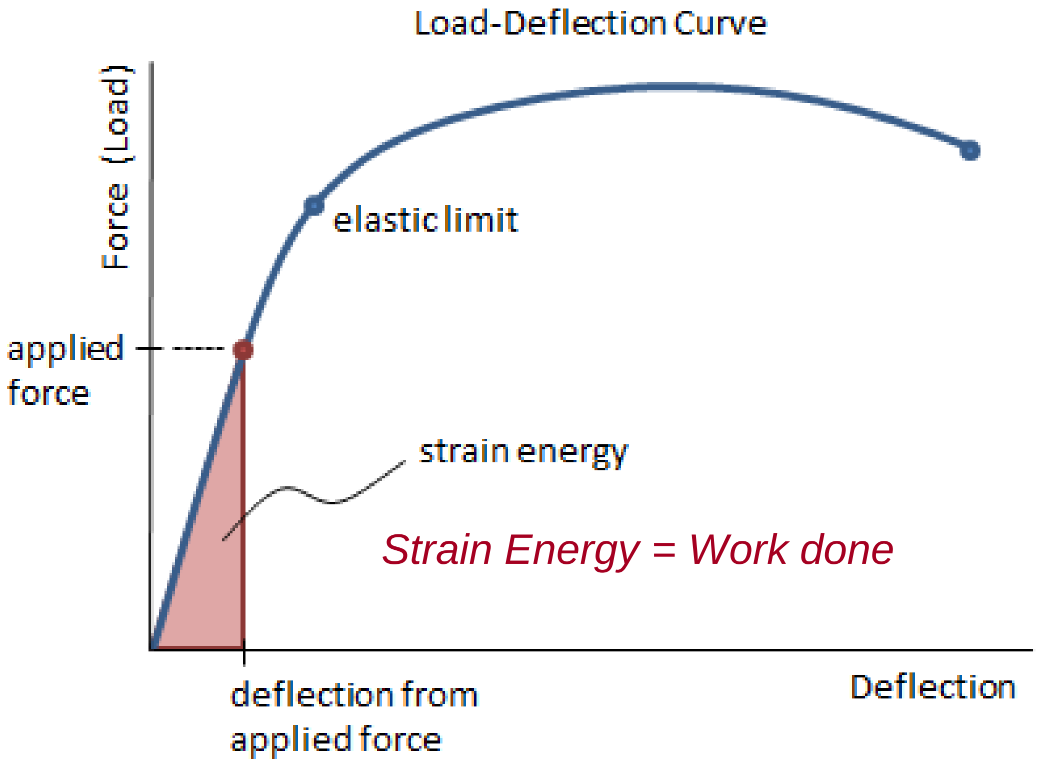
$$\gamma = \frac{\theta r}{L} = \frac{\tau}{G} = \frac{Tr}{GJ} \longrightarrow \theta = \frac{TL}{GJ} \longrightarrow \frac{T}{\theta} = k_t = \frac{GJ}{L}$$

Resistance of the element against torsional deflection

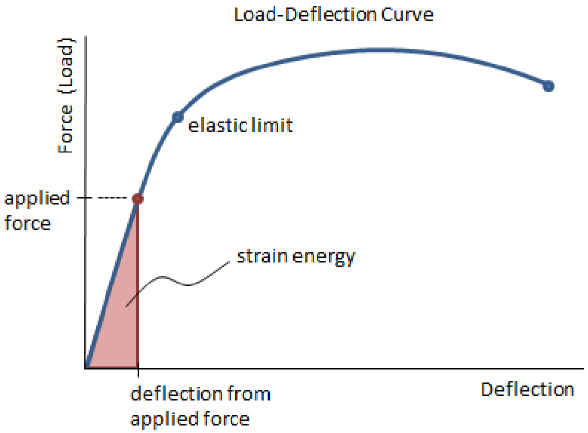
- There are many methods used for the determination of deflection of the elements. **Most known methods are:**
  - *Method of using Singularity Functions*
  - *Double Integration method*
  - *Numerical Integration method*
  - *Graphical Integration method*
  - *Area-Moment method*
  - ***Strain Energy method (Castigliano's Theorem)***
- In this course, **the emphasis is given to the application of strain energy method** because this is a powerful approach to solving a wide range of deflection analysis situations.

- **Strain energy** is the potential energy stored in an element due to elastic deformation caused by different loading conditions.
- Strain energy is dependent on the type of loading *(axial load, torque, bending moment, direct shear or transverse shear)* because the deformation due to the type of loading is different.
- It depends on;
  - The amount of load
  - Type of loading
  - Dimensions





- Let's look at the **strain energies** for all types of loading.



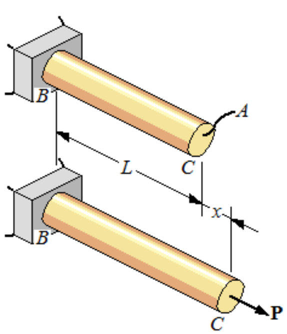
$$U = \frac{Fy}{2} \quad \text{and} \quad y = \frac{F}{k} \longrightarrow U = \frac{F^2}{2k}$$

This equation is general in the sense that the force F can also mean ***axial force****,* ***torque****, or* ***moment****.*

For axial loading

$$U = \frac{F^2}{2k} \quad \text{and} \quad k = \frac{AE}{L}$$

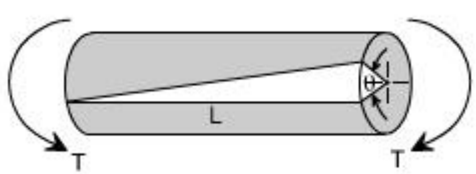
$$\Rightarrow U = \frac{F^2 L}{2AE}$$



For torsional loading

$$U = \frac{F^2}{2k_t} \quad \text{and} \quad k_t = \frac{GJ}{L}$$

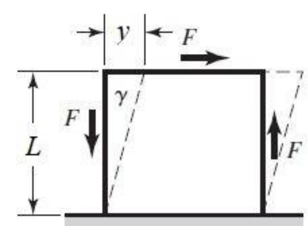
$$\Rightarrow U = \frac{T^2 L}{2GJ}$$



For direct shear

$$U = \frac{Fy}{2} \quad \text{and} \quad y = \frac{FL}{AG}$$

$$\Rightarrow U = \frac{F^2 L}{2AG}$$



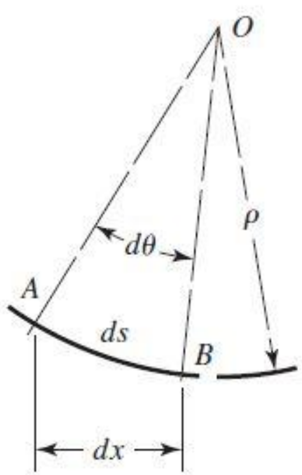
For bending moment

$$dU = \frac{M}{2} d\theta \quad \text{and} \quad \rho d\theta = ds \longrightarrow dU = \frac{Mds}{2\rho}$$

$$\frac{1}{\rho} = \frac{M}{EI} \longrightarrow dU = \frac{M^2 ds}{2EI}$$

For small deflections $\longrightarrow ds \cong dx$

$$U = \int \frac{M^2 dx}{2EI}$$



For transverse shear

Flexural shear may be subjected to change throughout the length of the element, therefore strain energy is expressed as;

$$U = \int \frac{CF^2 dx}{2GA}$$

C: 1.2 *for rectangular shape*
C: 1.11 *for circular shape*
C: 2.0 *for tubular shape*

Note: For most of the problems, the contribution of transverse shear on deflection is very small comparing the others and it is usually neglected.

- **As a summary:**

For axial loading ⟶ $U = \dfrac{F^2 L}{2AE}$

For torsional loading ⟶ $U = \dfrac{T^2 L}{2GJ}$

For direct shear ⟶ $U = \dfrac{F^2 L}{2AG}$

For bending moment ⟶ $U = \int \dfrac{M^2 dx}{2EI}$

For transverse shear ⟶ $U = \int \dfrac{CF^2 dx}{2GA}$ *(usually neglected)*

- **Castigliano's Theorem states that** deflection of a member *at the point of application* and *in the direction of a force* can be found **by taking partial derivative of the total strain energy** with respect to that force.

$$y_i = \frac{\partial U}{\partial F_i}$$

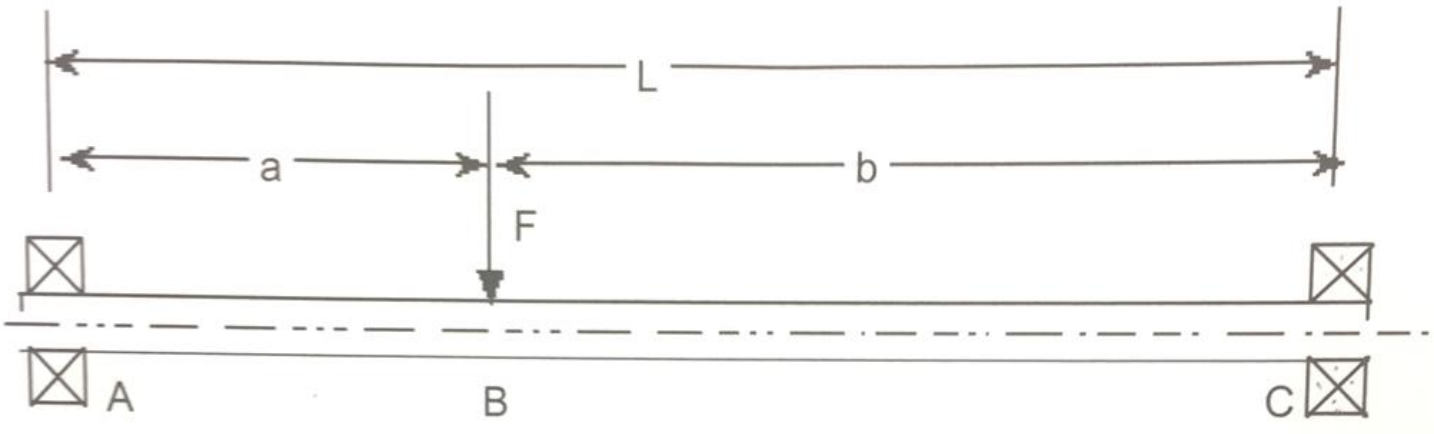
- **The slope of deflection curve** at the point of interest is obtained by taking derivative of total strain energy with respect to bending moment at that point.

$$\theta_i = \frac{\partial U}{\partial M_i}$$

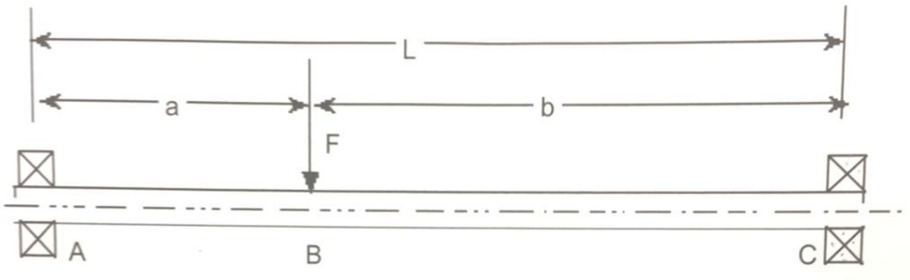
- For Castigliano's theorem to be applied **there must exist a concentrated load at the point under consideration**. If the deflection of a member is required at a point where **no concentrated load is acting**, an imaginary *(fictitious)* force Q is placed at that point and in the resulting deflection expression Q is set to zero.

$$y = \left(\frac{\partial U}{\partial Q}\right)_{Q=0}$$

Consider a simply supported beam as shown below. Using Castigliano's theorem, develop an expression for the deflection at point B.



Solution of Example 3.1



At point B, there is a concentrated load F. Hence, Castigliano's theorem can directly be applied to this case.

$$y_B = \frac{\partial U}{\partial F}$$

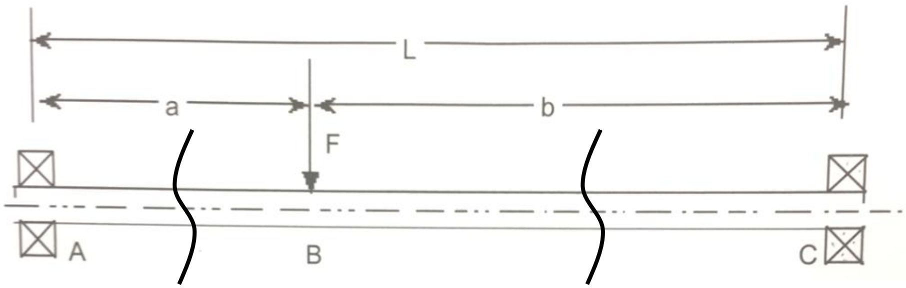
Where U is the total energy which is the sum of strain energies stored in parts AB and BC.

$$U = U_{AB} + U_{BC} \qquad U_{AB} = \int \frac{M_{AB}{}^2 dx}{2EI} \quad \text{and} \quad U_{BC} = \int \frac{M_{BC}{}^2 dx}{2EI}$$

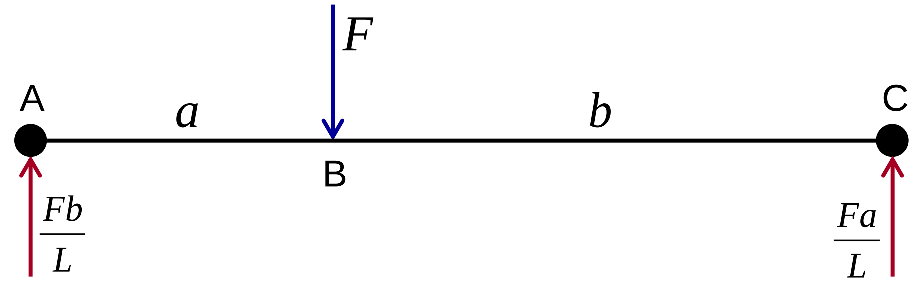
$$y_B = \frac{\partial U}{\partial F} = \frac{1}{EI}\left[\int_0^a M_{AB} \frac{\partial M_{AB}}{\partial F} dx + \int_a^L M_{BC} \frac{\partial M_{BC}}{\partial F} dx\right]$$

***M_{AB}, M_{BC}** and **partial derivatives of these expressions with respect to F** must be determined.*

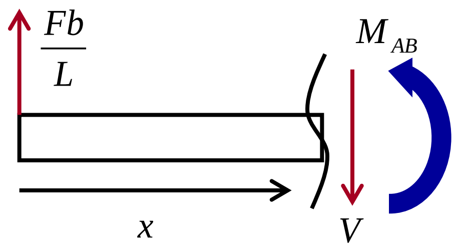
Solution of Example 3.1



Free Body Diagram



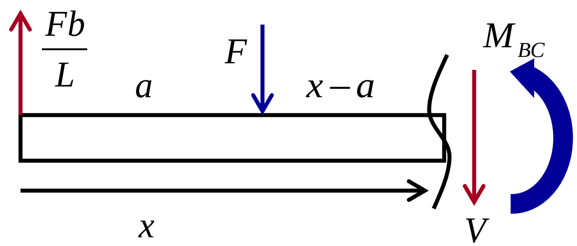
$0 < x \leq a$



$$M_{AB} = \frac{Fb}{L}x$$

$$\frac{\partial M_{AB}}{\partial F} = \frac{b}{L}x$$

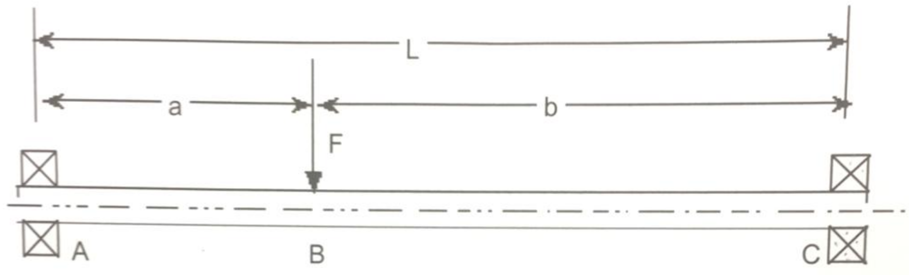
$a \leq x < L$



$$M_{BC} + F(x-a) = \frac{Fb}{L}x \longrightarrow M_{BC} = \frac{Fb}{L}x - F(x-a)$$

$$M_{BC} = \frac{Fb}{L}x - Fx + Fa \longrightarrow M_{BC} = F\left(\frac{b}{L} - 1\right)x + Fa$$

$$\frac{\partial M_{BC}}{\partial F} = \left(\frac{b}{L} - 1\right)x + a$$



$$M_{AB} = \frac{Fb}{L}x \qquad \frac{\partial M_{AB}}{\partial F} = \frac{b}{L}x \qquad M_{BC} = F\left(\frac{b}{L}-1\right)x + Fa \qquad \frac{\partial M_{BC}}{\partial F} = \left(\frac{b}{L}-1\right)x + a$$

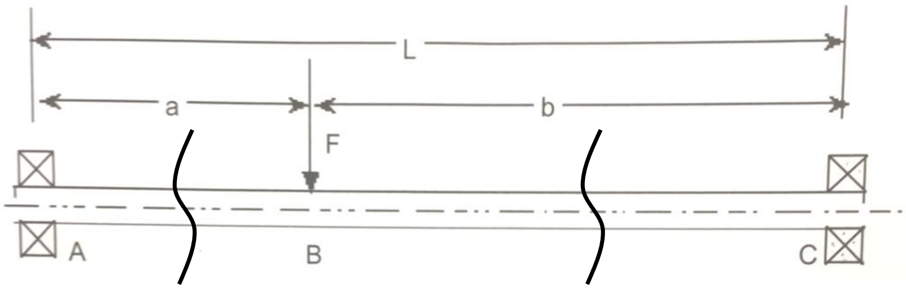
$$y_B = \frac{\partial U}{\partial F} = \frac{1}{EI}\left[\int_0^a M_{AB}\frac{\partial M_{AB}}{\partial F}dx + \int_a^L M_{BC}\frac{\partial M_{BC}}{\partial F}dx\right]$$

$$y_B = \frac{\partial U}{\partial F} = \frac{1}{EI}\left[\int_0^a \frac{Fb}{L}x\frac{b}{L}xdx + \int_a^L \left(F\left(\frac{b}{L}-1\right)x + Fa\right)\left(\left(\frac{b}{L}-1\right)x + a\right)dx\right]$$

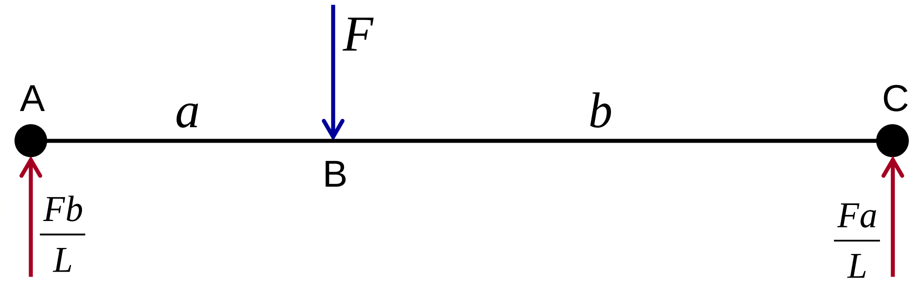
$$y_B = \frac{\partial U}{\partial F} = \frac{Fb^2a^3}{3EIL^2} + \frac{Fa^2b^3}{3EIL^2} = \frac{Fa^2b^2(a+b)}{3EIL^2} = \frac{a^2b^2}{3EIL}F$$

This problem can also be solved by writing moment expression M_{BC} by considering x is changing between 0 and b.

Solution of Example 3.1

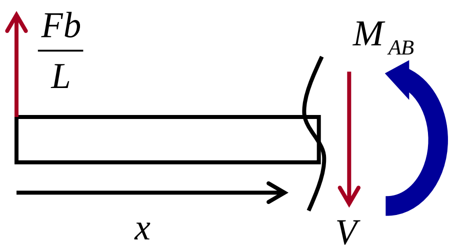


Free Body Diagram



Second Way

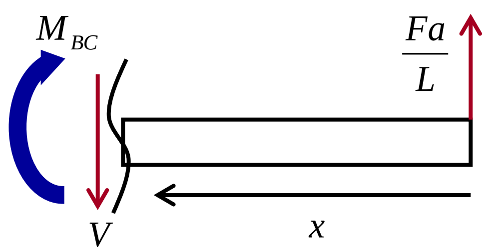
$0 < x \leq a$



$$M_{AB} = \frac{Fb}{L}x$$

$$\frac{\partial M_{AB}}{\partial F} = \frac{b}{L}x$$

$0 \leq x < b$



$$M_{BC} = \frac{Fa}{L}x$$

$$\frac{\partial M_{BC}}{\partial F} = \frac{a}{L}x$$

$$y_B = \frac{1}{EI}\left[\int_0^a M_{AB}\frac{\partial M_{AB}}{\partial F}dx + \int_0^b M_{BC}\frac{\partial M_{BC}}{\partial F}dx\right]$$

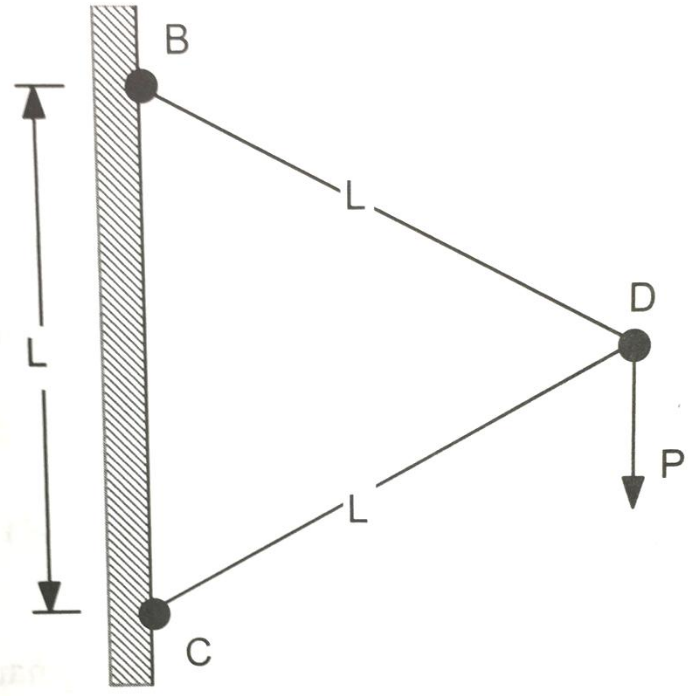
$$y_B = \frac{1}{EI}\left[\int_0^a \frac{Fb^2x^2}{L^2}dx + \int_0^b \frac{Fa^2x^2}{L^2}dx\right]$$

$$y_B = \frac{1}{EI}\left[\frac{Fb^2a^3}{3L^2} + \frac{Fa^2b^3}{3L^2}\right]$$

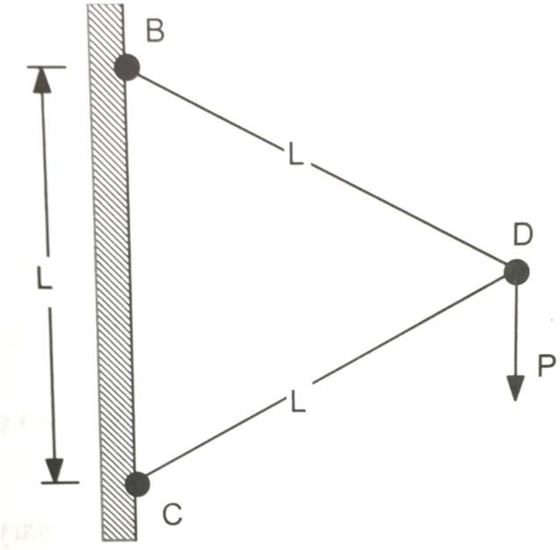
$$y_B = \frac{1}{EI}\left[\frac{Fa^2b^2(a+b)}{3L^2}\right] = \frac{Fa^2b^2}{3EIL}$$

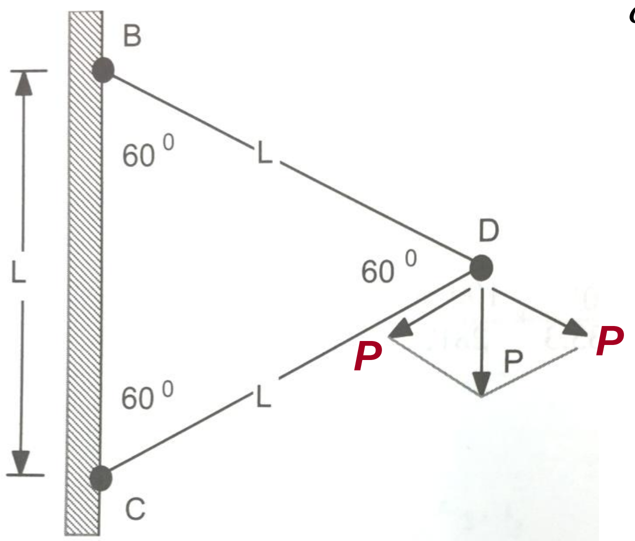
Example 3.3

A simple truss composed of two bars each of length *L* carries a vertical load *P* at joint D. Find the horizontal and vertical components of the total deflection of point D. The bars are made of same material, *DB having a cross sectional area of A* and *DC having a cross sectional area of 2A*. **Use theorem of Castigliano**.



Solution of Example 3.3

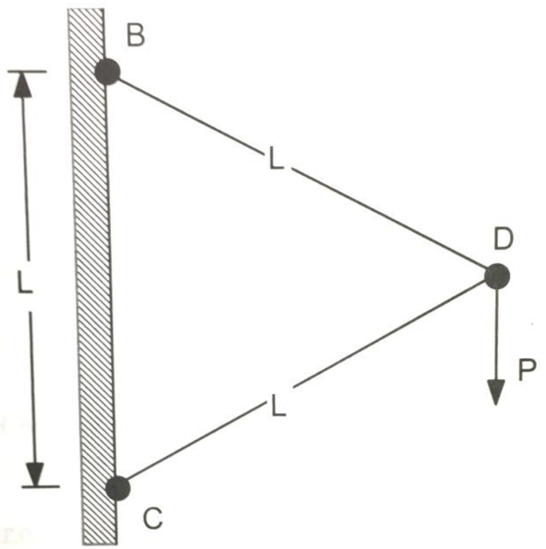




Vertical deflection will be in the direction of the external load P. As shown in Figure, BDC is an equilateral triangle. Force acting on bar DB and DC is equal to P.

$$U_{DB} = \frac{P^2 L}{2AE} \quad \textit{and} \quad U_{DC} = \frac{P^2 L}{4AE}$$

$$\delta_v = \frac{\partial\left(U_{DB} + U_{DB}\right)}{\partial P} = \frac{\partial}{\partial P}\left(\frac{P^2 L}{2AE}\right) + \frac{\partial}{\partial P}\left(\frac{P^2 L}{4AE}\right) = \frac{3}{2}\frac{PL}{AE}$$

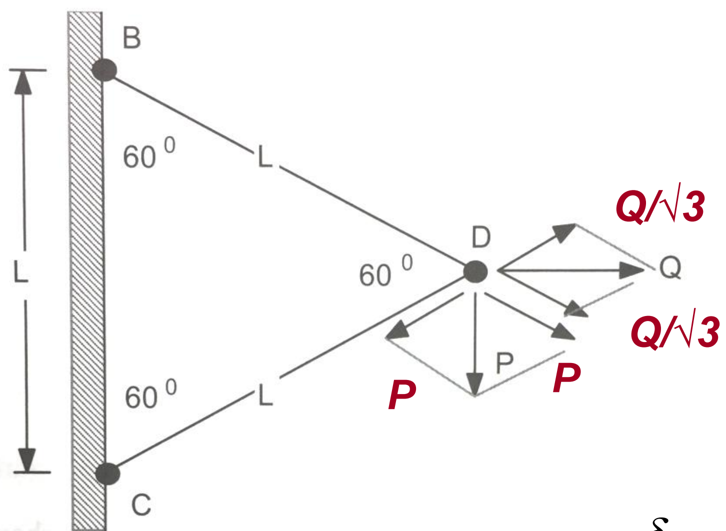


In order to determine the deflection in horizontal direction, we must apply an imaginary (fictitious) force Q at point D as shown in Figure.

Force on DB= $P+\frac{Q}{\sqrt{3}}$ **Force on DC=** $P-\frac{Q}{\sqrt{3}}$

Energies stored in these bars due to the addition of the imaginary force Q can be written as:

$$U_{DB}=\frac{\left(P+\frac{Q}{\sqrt{3}}\right)^2 L}{2AE} \qquad U_{DC}=\frac{\left(P-\frac{Q}{\sqrt{3}}\right)^2 L}{4AE}$$



$$\delta_h=\frac{\partial\left(U_{DB}+U_{DB}\right)}{\partial Q}=\frac{\partial}{\partial Q}\left(\frac{\left(P+\frac{Q}{\sqrt{3}}\right)^2 L}{2AE}+\frac{\left(P-\frac{Q}{\sqrt{3}}\right)^2 L}{4AE}\right)$$

$$\delta_h=\frac{\left(P+\frac{Q}{\sqrt{3}}\right)\frac{1}{\sqrt{3}}L}{AE}+\frac{\left(P-\frac{Q}{\sqrt{3}}\right)\left(-\frac{1}{\sqrt{3}}\right)L}{2AE}=\frac{PL}{2\sqrt{3}AE}$$

ME 307 – Machine Elements I

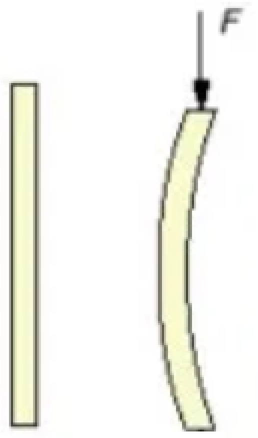
Chapter 3

Deflection Analysis (Part II)

Mechanical Engineering
University of Gaziantep

Dr. Sadık Olguner
Dr. Hakan Çandar

- **Buckling** is another consideration in the design of elements subjected to a compression load.
- If a bar is subjected to a compression by a force acting along the direction of centroidal axis, it will shorten according to Hooke's Law.
- If the magnitude of the the force is increased to a value at which the stress greater than the yield strength of the material, then the material is squeezed into a flat disc or it fractures.
- If the bar is long or length to diameter ratio is large, as the compression load increases, a critical value will be reached corresponding to an unstable condition.
- At this stage, any slight increase or movement in the load will result a sudden and total collapse of the bar.
- This failure is known as **buckling failure** and the load at this condition is called as the **critical load**.



- **If there is a compression type of loading,** in addition to compression stress, **buckling failure should also be checked!!!**

- **Critical Load** is found from:

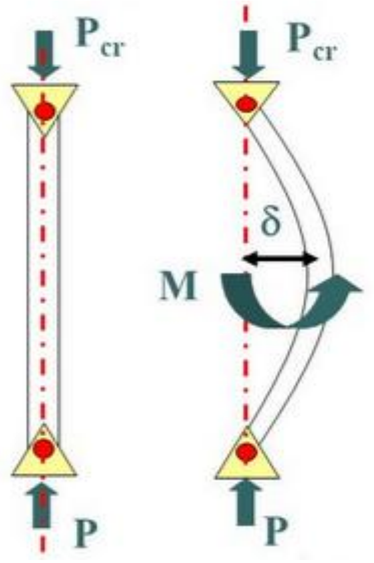
$$P_{cr} = \frac{C\pi^2 EI}{L^2}$$

- Sometimes it is more convenient to use **Critical Unit Load** which is defined as:

$$\frac{P_{cr}}{A} = \frac{C\pi^2 E}{(L/k)^2}$$

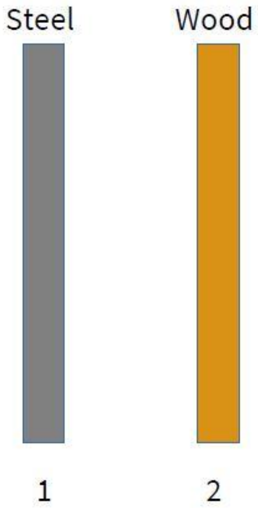
***C** is the constant depends on the support condition.*

- This expression is coming from the bending deflection equation of the column.



Parameters Influence Buckling Failure

➤ Modulus of Elasticity (E)

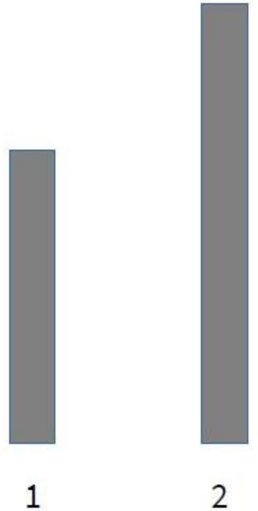


$$P_{cr} = \frac{C\pi^2 EI}{L^2}$$

-Same length
-Same shape
-Same area
-Different material (Steel & wood)

-$P_{cr1} > P_{cr2}$

➤ Length (L)

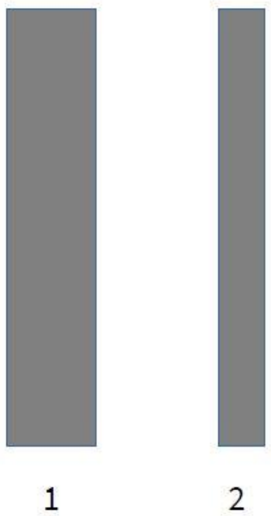


$$P_{cr} = \frac{C\pi^2 EI}{L^2}$$

-Same material
-Same shape
-Same area
-Length is different

-$P_{cr1} > P_{cr2}$

Area (A)

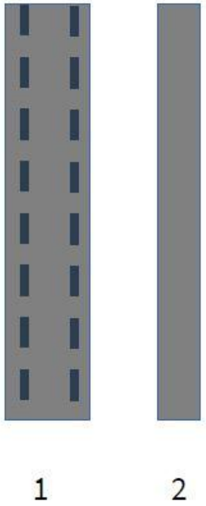


$$P_{cr} = \frac{C\pi^2 EI}{L^2}$$

-Same material
-Same shape
-Same length
-Area is different

-P_{cr1} > P_{cr2}

Area Moment of Inertia (I)

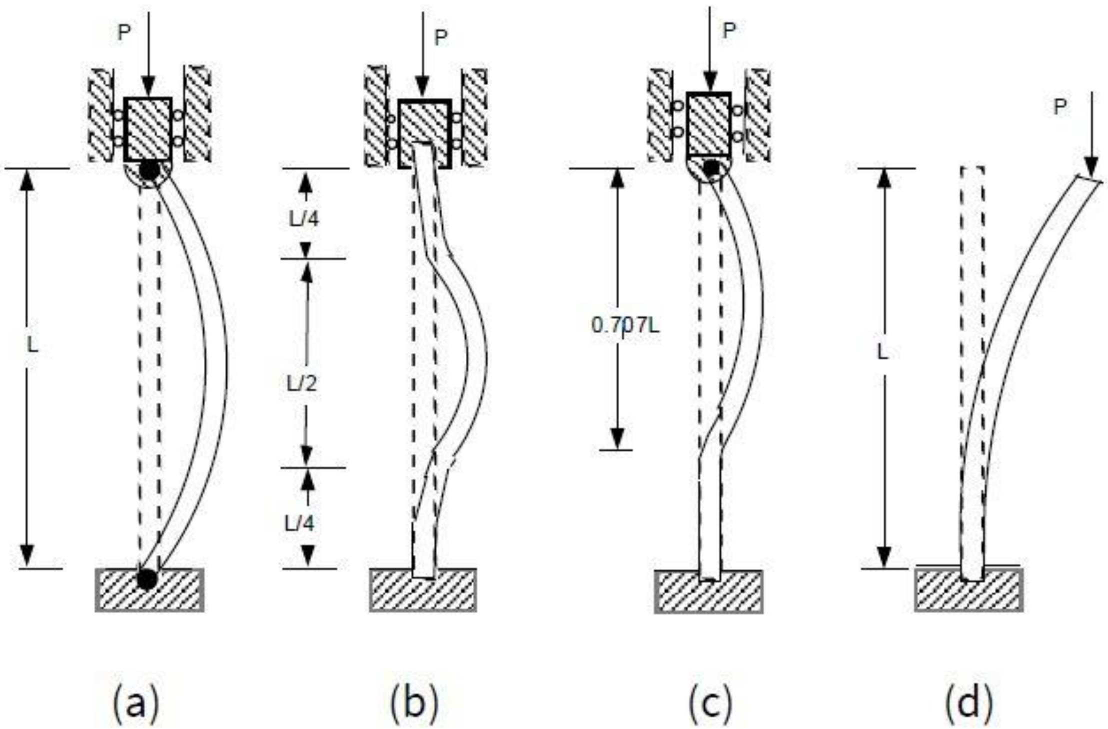


$$P_{cr} = \frac{C\pi^2 EI}{L^2}$$

-Same material
-Same area
-Same length
-Shape is different

-P_{cr1} > P_{cr2}

- In the determination of the Critical Load, end conditions (end fixity) ***"C"*** must also always be taken into account. One can mention about four types of end conditions:



(a) pinned ends, C=1

(b) fixed ends, C=4

(c) pinned&fixed ends, C=2

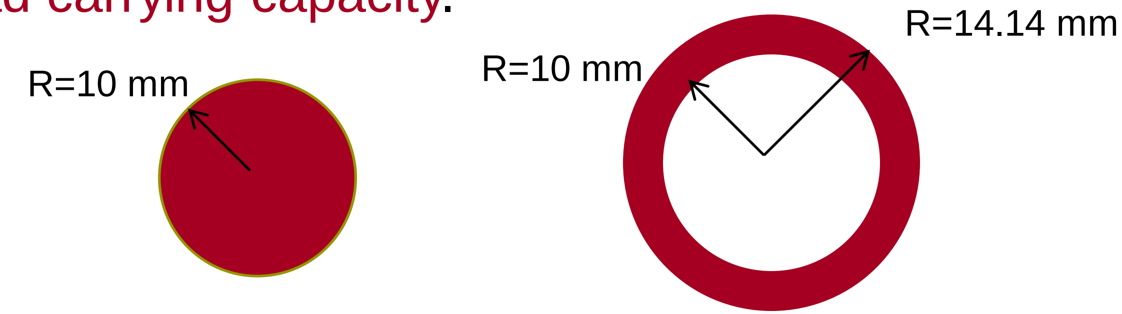
(d) fixed&free ends, C=1/4

C is the constant depends on the support condition

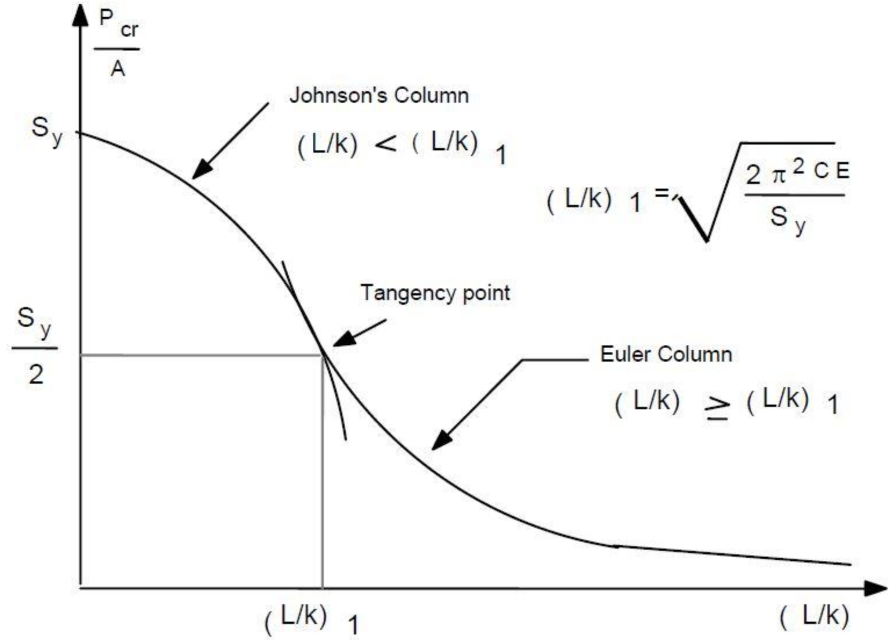
- **Slenderness Ratio** is described as length to radius of gyration. **SR = L/k**
- **Radius of gyration** can be defined as a measure of distribution of area about centroidal axis and it is expressed as:

$$k = \sqrt{\frac{I}{A}}$$

- For example; **solid bar** with a diameter of 20 mm has cylindrical cross sectional area of 314 mm^2 and radius of gyration of k=d/4=5 mm. **A hollow cylinder** which has the same area (an inner diameter of 20 mm and outside diameter of 28.28 mm) has the radius of gyration of $5(3)^{1/2}$=8.66 mm. This higher value of **k** indicates higher compressive load carrying capacity.



- For concentric loading, variation of critical unit load with respect to slenderness ratio (L/k) is illustrated in the figure.
- The figure indicates that columns exhibit different characteristics below and above of a special value of slenderness ratio $(L/k)_1$.



- This special value of slenderness ratio is used for justifying whether the column is **Euler** or **Johnson's** column. It is expressed as:

$$\left(\frac{L}{k}\right)_1 = \sqrt{\frac{2\pi^2 CE}{S_y}}$$

- If **(L/k)** of the column is greater than or equal to $(L/k)_1$ then the column is considered to be an **Euler column**. If **(L/k)** is less than $(L/k)_1$ then the column is considered to be a **Johnson's column.**

- JOHNSON (short columb)

$(L/k) < (L/k)_1$

$$\frac{P_{cr}}{A} = S_y - \left(\frac{S_y}{2\pi}\right)^2 \left(\frac{1}{CE}\right)\left(\frac{L}{k}\right)^2$$

- EULER (long columb)

$(L/k) > (L/k)_1$

$$P_{cr} = \frac{C\pi^2 EI}{L^2}$$

Critical load

$$\frac{P_{cr}}{A} = \frac{C\pi^2 E}{(L/k)^2}$$

Unit critical load

1. Calculate the slenderness ratio; (L/k)

2. Calculate $(L/k)_1$ from the following formulation;

$$\left(\frac{L}{k}\right)_1 = \sqrt{\frac{2\pi^2 CE}{S_y}}$$

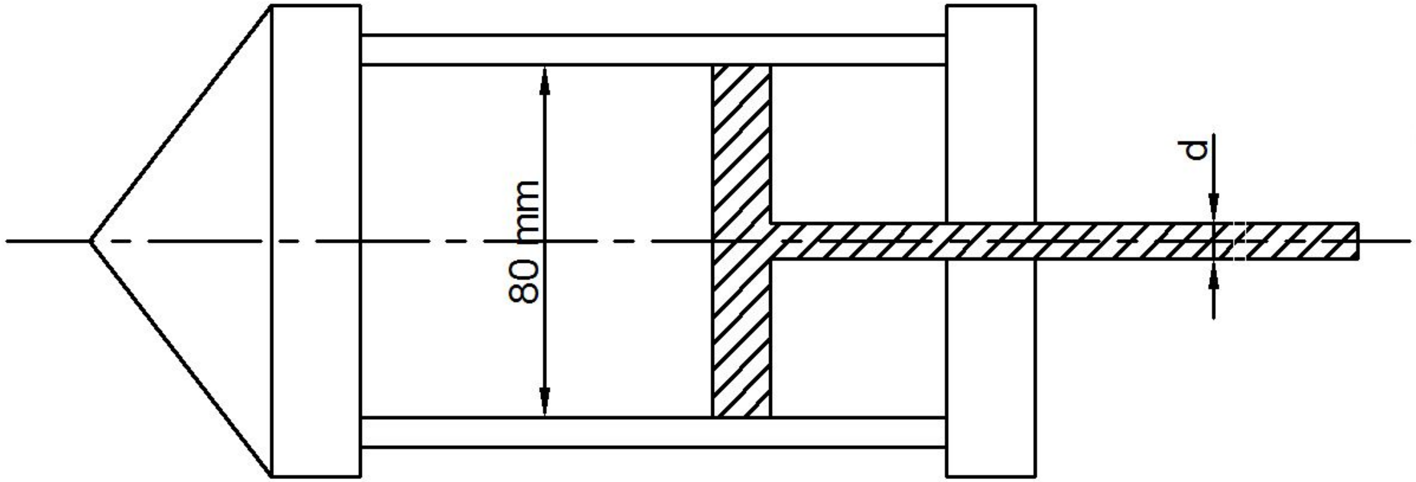
3. Check if $(L/k)<(L/k)_1$ or $(L/k)>(L/k)_1$

Johnson's — Euler

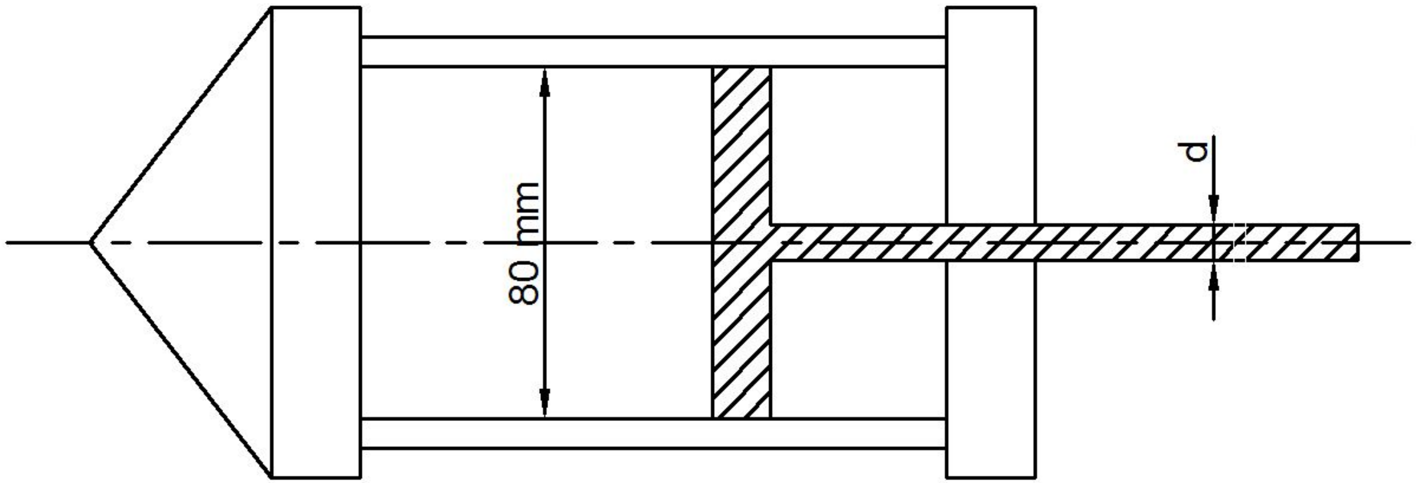
Use relevent formulation to calculate P_{cr}

4. Check if $P<P_{cr}$, if yes, safe for buckling

- The hydraulic cylinder operates 5.6 MPa pressure. The piston and rod should be sized as **a column with both ends rounded** for any plane of buckling. Select a preffered size of the rod diameter if the column length is 1.5 m. Sy=260 Mpa.



Solution



$$P = \frac{F}{A} \Rightarrow 5.6 = \frac{4F}{\pi(80)^2} \Rightarrow F = 28149N$$

For compression stress:

$$\sigma = \frac{F}{A} \Rightarrow 260 = \frac{28149}{A} \Rightarrow A \cong 108mm^2$$

$$A \cong 108mm^2 = \frac{\pi d^2}{4} \Rightarrow d = 11.7 \cong 12mm$$

Assume it is an Euler Column. Due to both ends are rounded, ***C=1***

$$P_{cr} = \frac{\pi^2 EI}{L^2} \Rightarrow I = \frac{FL^2}{\pi^2 E}$$ *and* $$I = \frac{\pi}{64} d^4$$

$$I = \frac{\pi}{64} d^4 = \frac{28149(1500)^2}{\pi^2 (207000)}$$

$d = 28.2mm$ *say* $d = 30mm$

Check if it is an Euler column or not.

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}d^4}{\frac{\pi}{4}d^2}} = \frac{d}{4} = \frac{30}{4} = 7.5mm$$

Slenderness ratio: $\frac{L}{k} = \frac{1500}{7.5} = 200$

$$\left(\frac{L}{k}\right)_1 = \sqrt{\frac{2\pi^2 CE}{Sy}} = \sqrt{\frac{2\pi^2(1)(207000)}{260}} = 125.36$$

$$\frac{L}{k} \geq \left(\frac{L}{k}\right)_1$$

It is an Euler column so the assumption is correct.

D = 30 mm.

ME 307 – Machine Elements I

Chapter 4

Design for Static Strength

Mechanical Engineering
University of Gaziantep

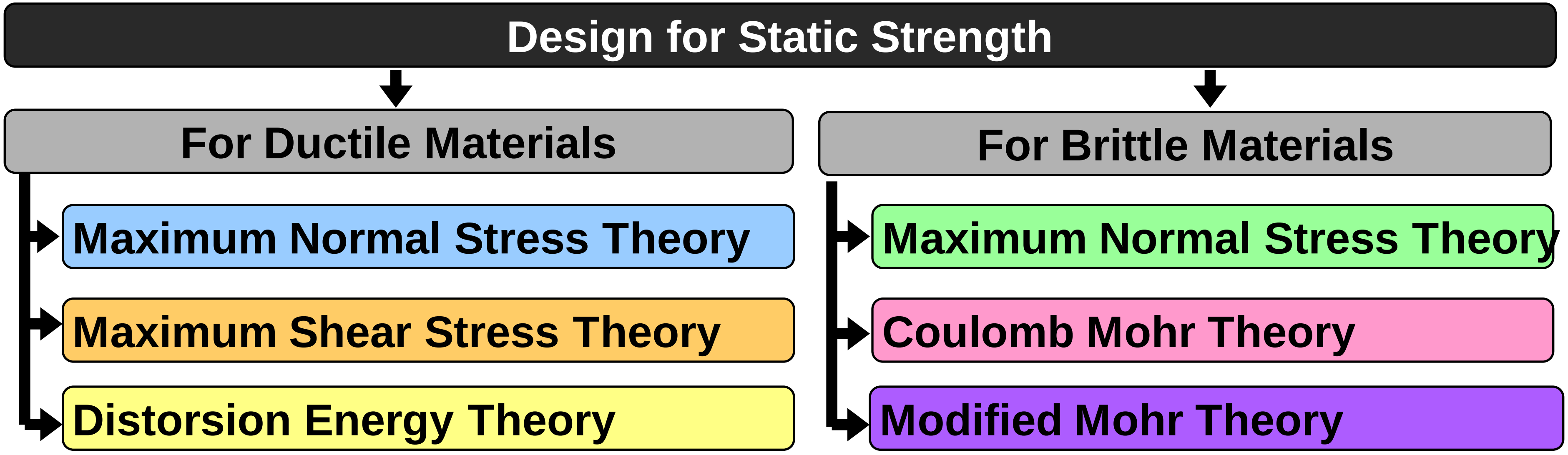
Dr. Sadık Olguner
Dr. Hakan Çandar

- **Strength** is a characteristic of the material such that **it is independent from loading**. Strength of an element is **dependent on the type, treatment** and **processing** of the material.
- The letter "***S***" is used to denote the strength. Type of strength is differentiated with the subscript. *(**S_{ut}** indicates the ultimate tensile strength, **S_{yt}** indicates tensile yield strength)*
- A system is described as a **static system if its state is not changing by time**. If the **magnitude**, **direction** and **point of application** of the load are not changing by time than loading condition is called as **static**.
- The main purpose of design for static strength is **to determine the size and/or to select the material of the machine element** which will function properly for the resulting static stress state.
- Sizing of the element must be arranged in such a way that **critical condition should not be reached.** *(permissible stress must not exceed material strength)*.

- In design studies, the first step is the **analysis of loading**.
- **Shear force** and **bending moment diagrams** must be drawn.
- **Torque** and **axial load distribution** throughout the system must also be evaluated.
- **Resulting stresses** are to be determined afterwards.
- Both **shear** and **normal stresses** may be acting on the worstly stressed element.
- **Failure** will occur when stresses exceed the material strength.
- The question is how we are going to compare these stresses with strength of material. Is there any available tool proposed for this purpose?

The answer is YES!

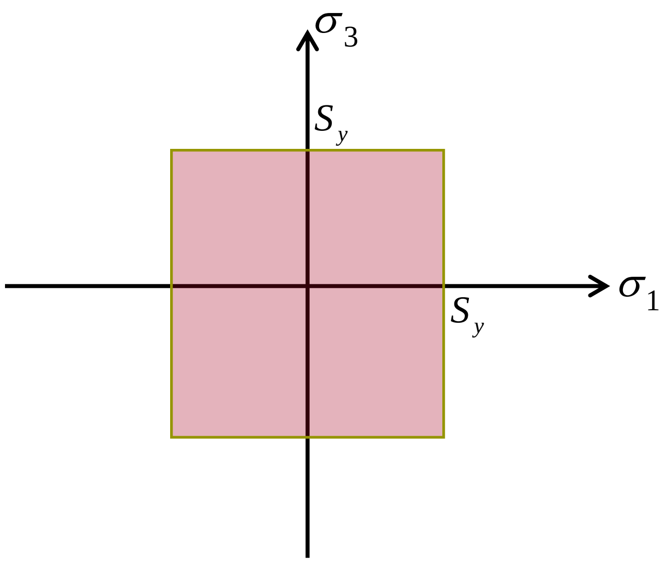
- In the case of ductile materials, **yield strength** is taken as **material strength** because yielding is considered to be the failure. ***($S_{yt} = S_{yc}$)***
- If the material is of brittle type, there will be no yield point and **failure would be a sudden fracture**. **Ultimate strength** is to be used as the **criteria** in the design process. ***($S_{uc} > S_{ut}$)***
- Since ductile and brittle materials exhibits different characteristic in failure, **failure theories will also be different**.



Maximum Normal Stress Theory of Failure

- Maximum normal stress theory of failure **(MNST)** states that failure will occur **when the maximum normal stress** *(largest principle stress)* in the element is equal or greater than the **normal stress in a tensile test specimen at the yield point**.
- If the principal stresses are arranged as $\boldsymbol{\sigma_1>\sigma_2>\sigma_3}$, then failure will occur when $\boldsymbol{\sigma_1 \geq S_y}$

Factor of safety $n = \dfrac{S_y}{\sigma_1}$



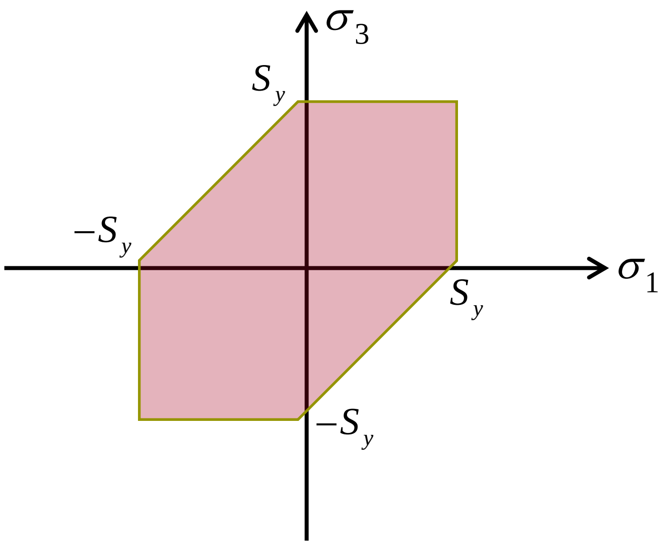
Maximum Shear Stress Theory of Failure

- Maximum shear stress theory of failure **(MSST)** *(also known as **Tresca** yield criterion)* states that failure will occur **when the maximum shear stress** in the element is equal or greater than the **shear stress in a tensile test specimen at the yield point**.

Maximum shear stress for biaxial stress state $$\tau_{max} = \frac{(\sigma_1 - \sigma_3)}{2}$$

Maximum shear stress in a tensile test specimen at the yield point $$S_{sy} = \frac{\sigma_1}{2} = \frac{S_y}{2}$$

Factor of safety $$n = \frac{S_{sy}}{\tau_{max}} = \frac{S_y}{(\sigma_1 - \sigma_3)}$$



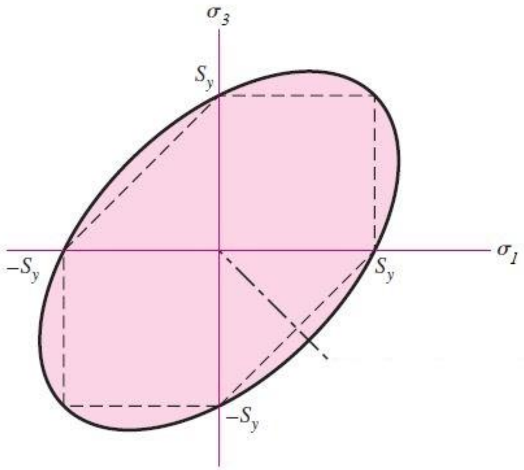
Distortion Energy Theory of Failure

- Distortion energy theory of failure **(DET)** *(also known as **Von-Mises** theory)* states that an element will fail **when the distortion energy in that element** is equal or greater than the **distortion energy in a tensile test specimen at the yield point**.

Effective or Von-Mises stress
$$\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

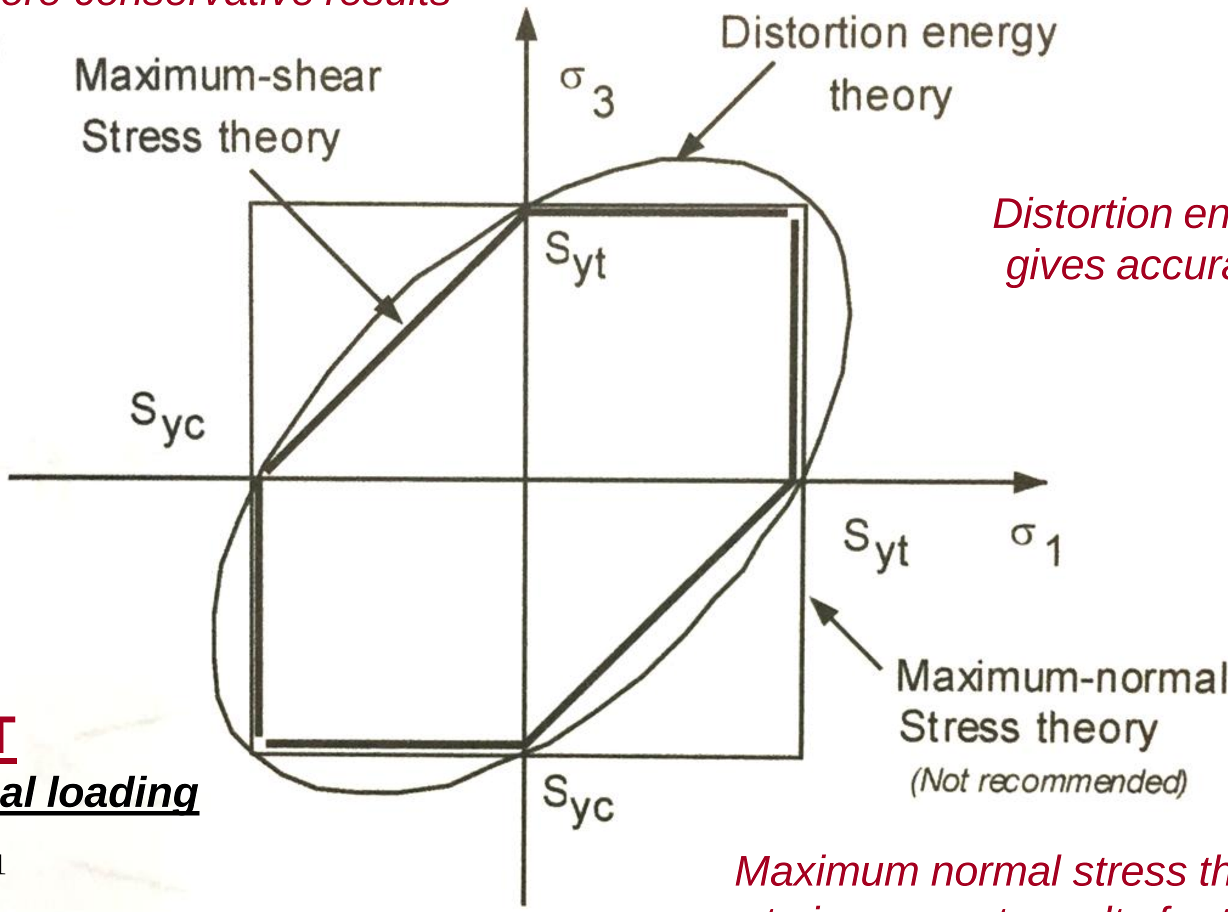
For biaxial stress state
$$\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2} \quad \textit{or} \quad \sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

Factor of safety
$$n = \frac{S_y}{\sigma'}$$



Failure Theories for Ductile Materials

Maximum shear stress theory gives more conservative results



Distortion energy theory gives accurate results.

MNST

for pure torsional loading

$$\tau_{\max} = \sigma_1$$

failure occurs when

$$\tau_{\max} = S_y$$

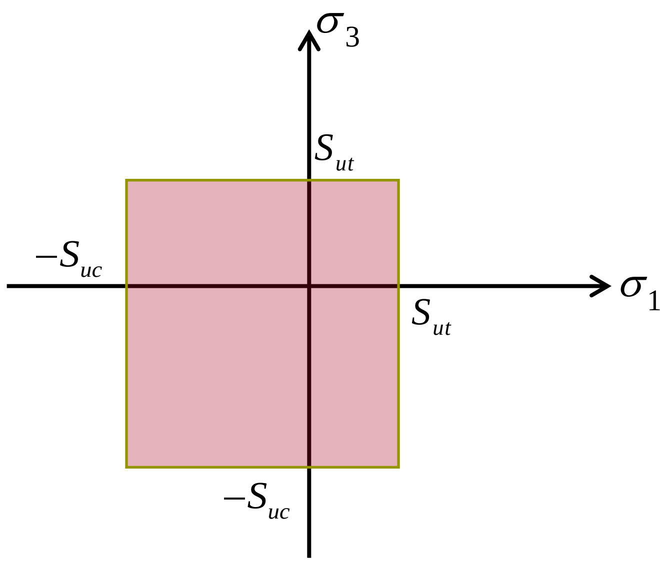
which is not true!

Maximum normal stress theory does not give correct results for the element subjected to pure torsional loads.

Maximum Normal Stress Theory of Failure

- Maximum normal stress theory of failure **(MNST)** states that failure will occur **when the maximum normal stress** *(largest principle stress)* in the element is equal or greater than the **normal stress in a tensile test specimen at the yield point**.
- If the principal stresses are arranged as $\sigma_1 > \sigma_2 > \sigma_3$, then failure will occur when $\sigma_1 \geq S_{ut}$ *or* $\sigma_3 \geq S_{uc}$

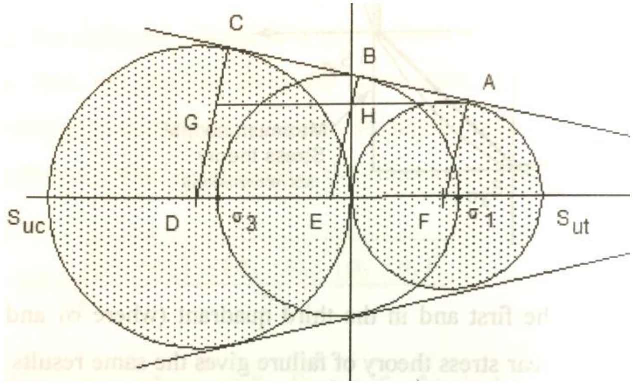
Factor of safety $n = \dfrac{S_{ut}}{\sigma_1}$ *or* $n = \dfrac{S_{uc}}{\sigma_3}$



Coulomb Mohr Theory of Failure

- Coulomb Mohr theory **(CMT)** *(also known as **Internal Friction Theory**)* states that failure will occur when **the stress state** on an element results in a mohr's circle which is tangent to the common tangent of mohr's circle when that element is loaded by a tensile and compression load seperately.

$$n = \frac{S_3}{\sigma_3} \quad \textit{where} \quad S_3 = \frac{S_{uc}}{\frac{S_{uc}}{S_{ut}}\frac{\sigma_1}{\sigma_3} - 1}$$

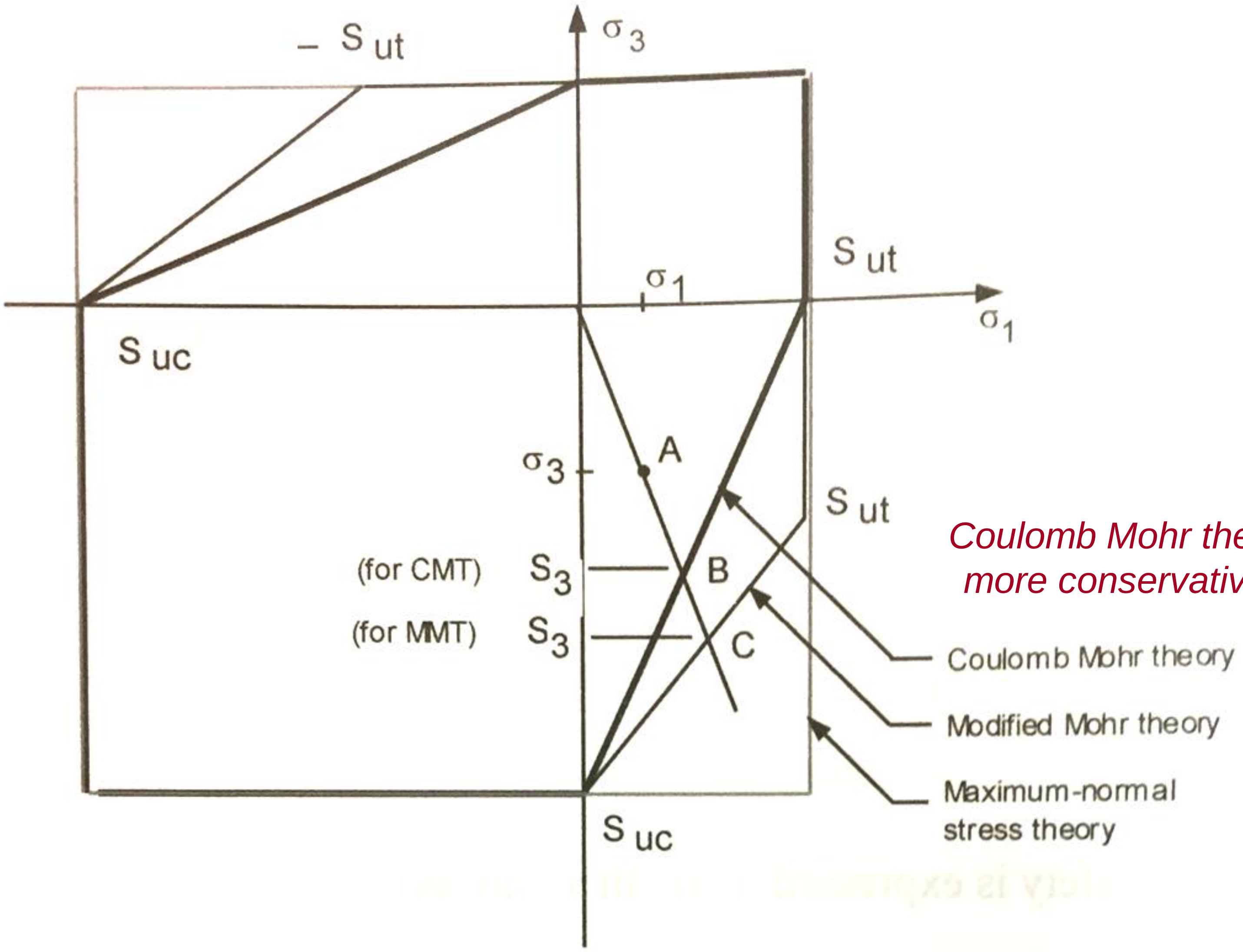


The Modified Mohr Theory of Failure

- The Modified Mohr theory **(MMT)** is different from the Coulomb Mohr theory in the fourth quadrant (σ_1 , $-\sigma_3$). In this quadrant, Modified Mohr theory is the same as maximum normal stress theory until $\boldsymbol{\sigma_3}$ becomes very near to $\boldsymbol{-S_{ut}}$

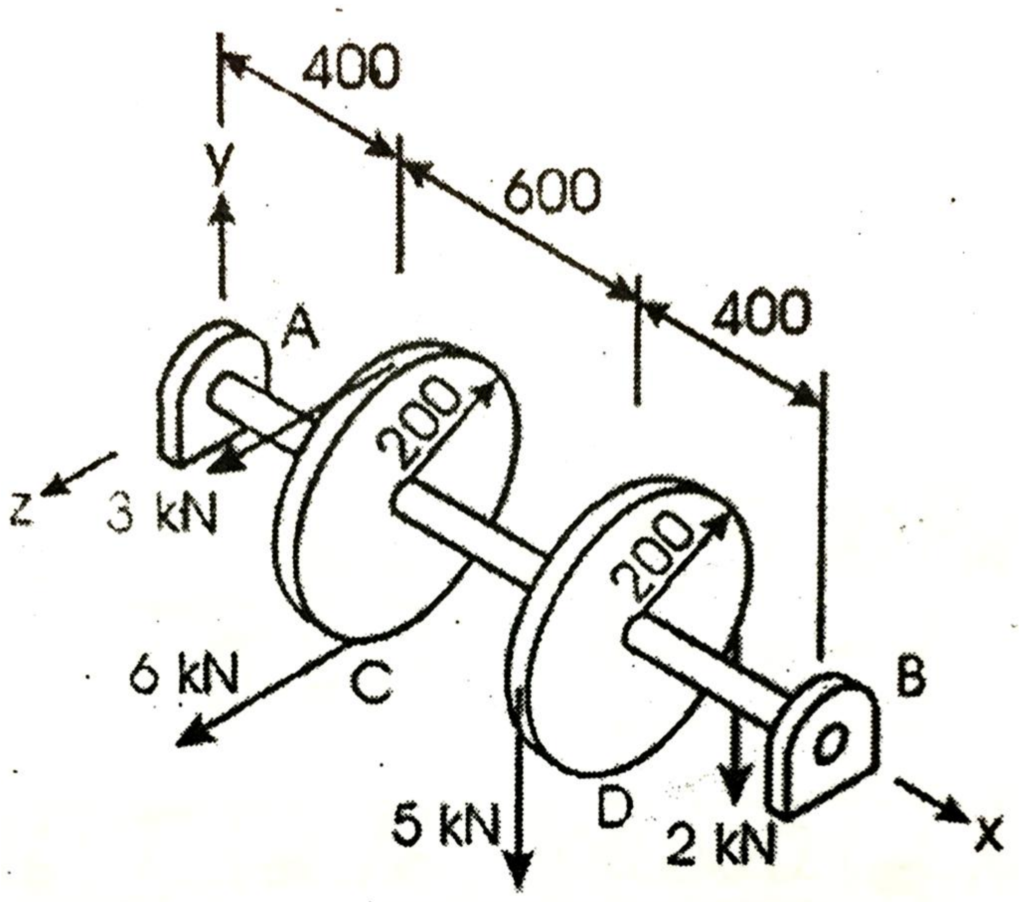
$$n = \frac{S_3}{\sigma_3} \quad \textit{where} \quad S_3 = \frac{S_{uc}}{\frac{S_{uc} - S_{ut}}{S_{ut}}\frac{\sigma_1}{\sigma_3} - 1}$$

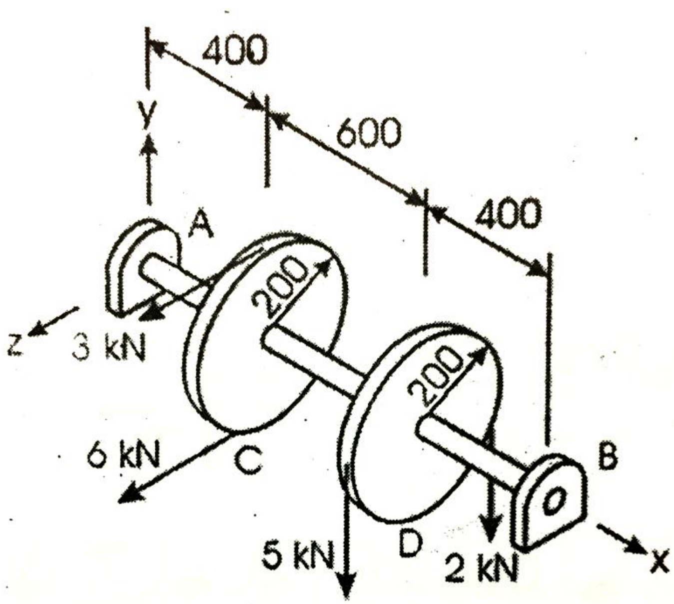
Failure Theories for Brittle Materials



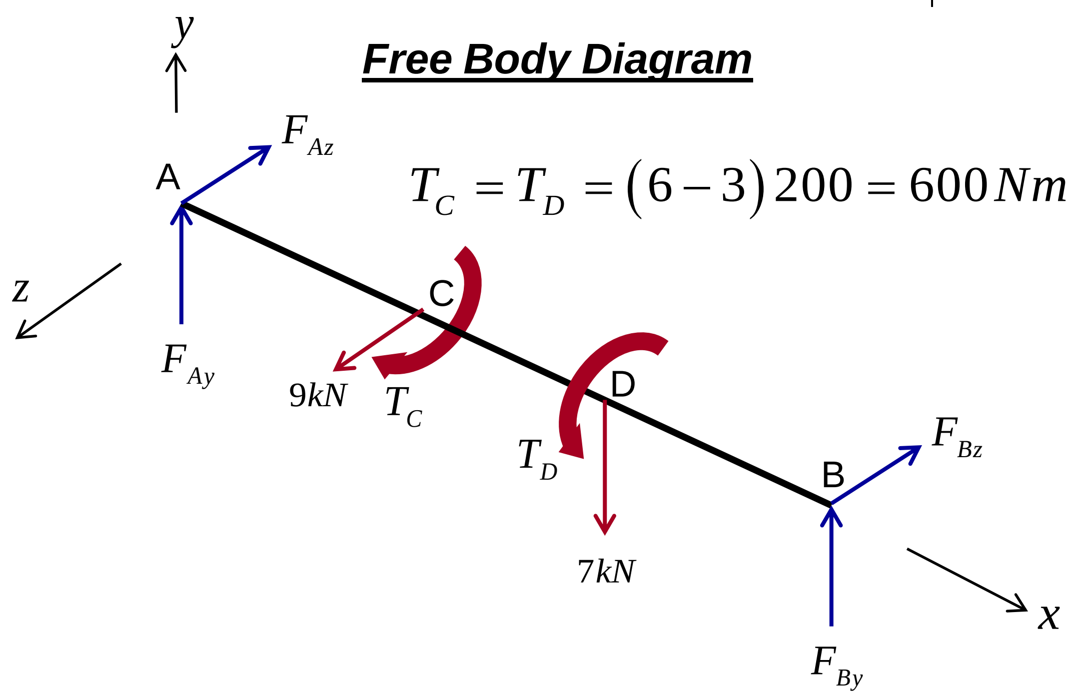
Coulomb Mohr theory gives more conservative results

The figure shows a belt pulley mechanism which is loaded statically. The shaft is made of AISI 1030 steel with the yield strength of 480 MPa. Using Distortion Energy Theory (DET), determine the diameter of the shaft with a factor of safety of 2. *(all dimensions are in mm.)*



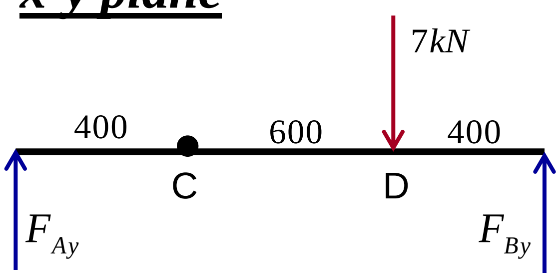


Free Body Diagram



$$T_C = T_D = (6-3)\,200 = 600\,Nm$$

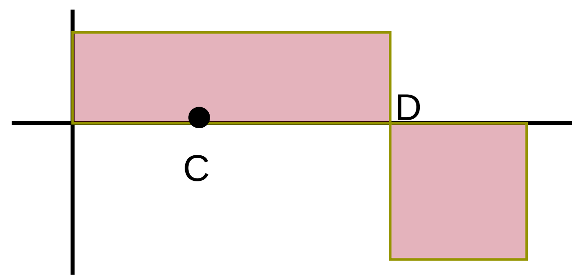
x-y plane

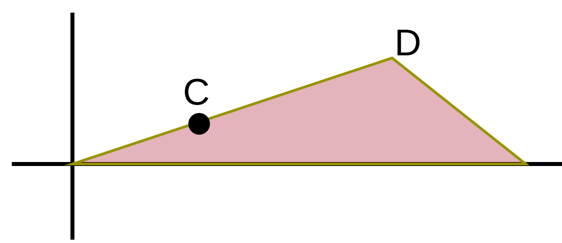


$$\sum M_A = 0 \Rightarrow 7*1000 - F_{By}*1400 = 0$$

$$F_{By} = 5kN$$

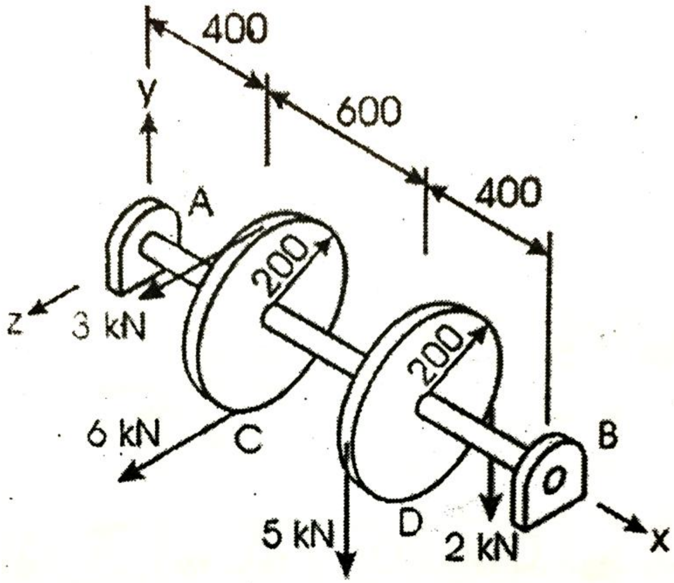
$$\sum F_y = 0 \Rightarrow F_{Ay} + 5 - 7 = 0 \Rightarrow F_{Ay} = 2kN$$



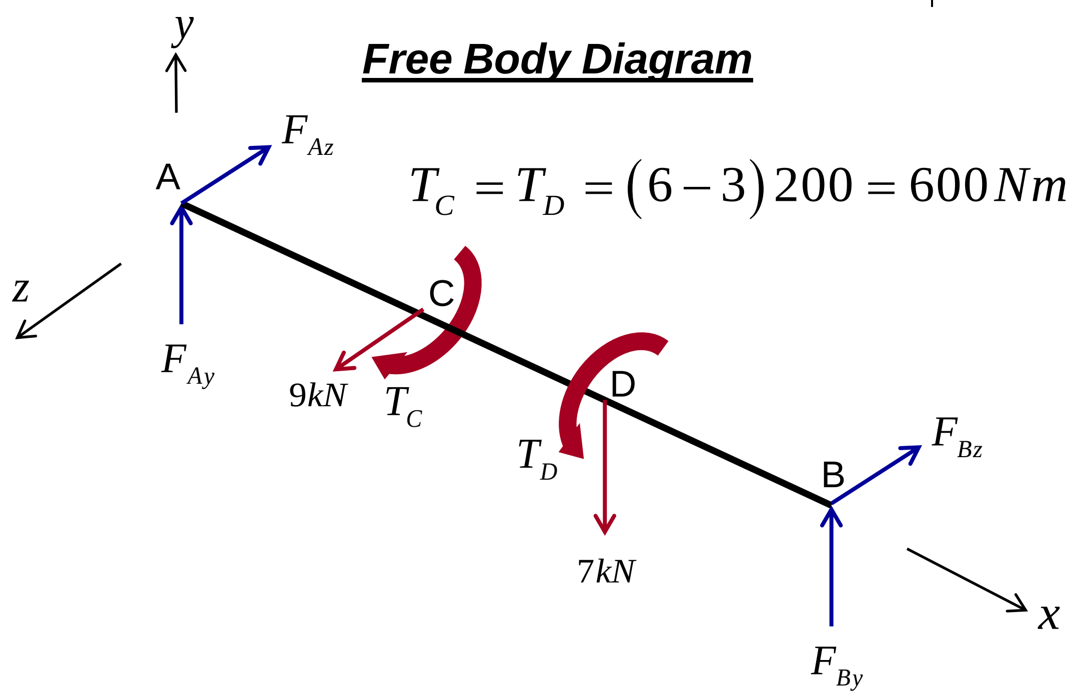


$$M_{Cz} = 800\,Nm$$

$$M_{Dz} = 2000\,Nm$$

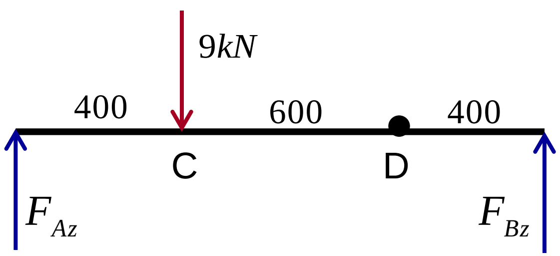


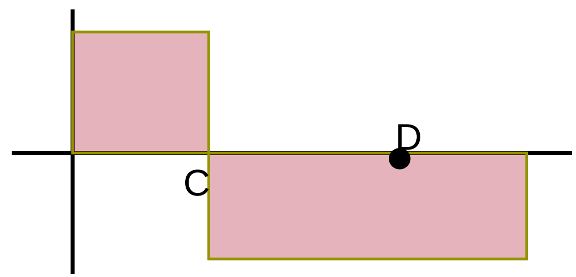
Free Body Diagram



$$T_C = T_D = (6-3)200 = 600\,Nm$$

x-z plane

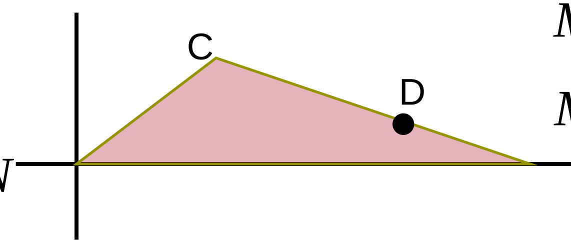




$$\sum M_A = 0 \Rightarrow 9*400 - F_{Bz}*1400 = 0$$

$$F_{Bz} = 2.57kN$$

$$\sum F_z = 0 \Rightarrow 9 - F_{Az} - 2.57 = 0 \Rightarrow F_{Az} = 6.43kN$$

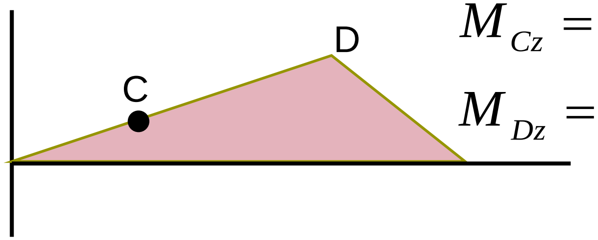


$$M_{Cy} = 2572\,Nm$$

$$M_{Dy} = 1028\,Nm$$

Solution of Example

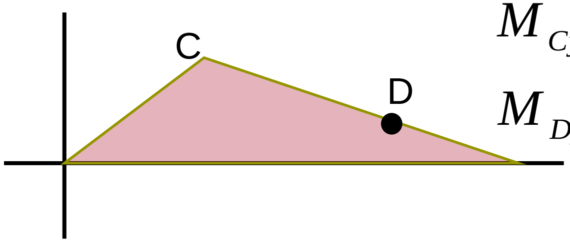
x-y plane



$$M_{Cz} = 800\,Nm$$

$$M_{Dz} = 2000\,Nm$$

x-z plane



$$M_{Cy} = 2572\,Nm$$

$$M_{Dy} = 1028\,Nm$$

From bending moment diagrams it is seen that **section C and D are two critical sections** where bending moments are maximum. Therefore design will be based on **either Section C or Section D**. We need to calculate resultant moments to define critical section.

$$M_C = \sqrt{M_{Cz}^2 + M_{Cy}^2} = \sqrt{800^2 + 2572^2} = 2693.5\,Nm \quad \textbf{(CRITICAL)}$$

$$M_D = \sqrt{M_{Dz}^2 + M_{Dy}^2} = \sqrt{2000^2 + 1028^2} = 2248.7\,Nm$$

It is seen that **point C is more critical**. Because diameters on both sections are same and same torque is acting for both sections. Thus, **maximum stress will occur at section C**.

Stresses at Section C

$$\sigma_b = \frac{32M_C}{\pi d^3} = \frac{32(2693.5)(1000)}{\pi d^3} = \frac{27435765.7}{d^3}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(600)(1000)}{\pi d^3} = \frac{3055775}{d^3}$$

Assume that d=40 mm

$$\sigma_b = 428.6MPa$$

$$\tau_{xy} = 47.7MPa$$

Distortion Energy Theory (DET)

$$\sigma' = \sqrt{\sigma_x^{\ 2} + 3\tau_{xy}^{\ 2}}$$

$$\sigma' = \sqrt{(428.6)^2 + 3(47.7)^2} = 436.5MPa$$

$$n = \frac{S_y}{\sigma'} = \frac{480}{436.5} = 1.1 < 2$$ ***(UNSAFE)***

Assume that d=50 mm

$$\sigma_b = 219.4MPa$$

$$\tau_{xy} = 24.4MPa$$

Distortion Energy Theory (DET)

$$\sigma' = \sqrt{\sigma_x^{\ 2} + 3\tau_{xy}^{\ 2}}$$

$$\sigma' = \sqrt{(219.4)^2 + 3(24.4)^2} = 223.4MPa$$

$$n = \frac{S_y}{\sigma'} = \frac{480}{223.4} = 2.14 > 2$$ ***(SAFE)***

ME 307 – Machine Elements I

Chapter 5

Design for Fatigue Strength

Mechanical Engineering
University of Gaziantep

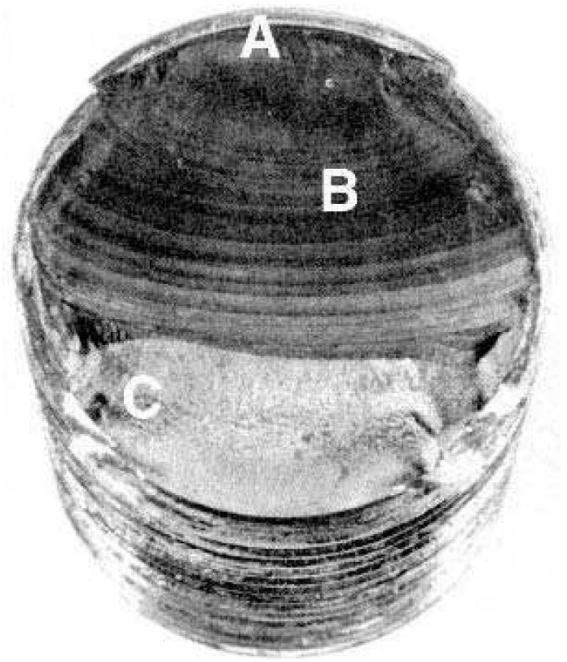
Dr. Sadık Olguner
Dr. Hakan Çandar

- **In the static design**, the system is **motionless**, and loading condition on the system **does not change with respect to time**.
  - *For a ductile material*, if the **equivalent stress** of the element is greater than the **yield strength** of the material, **failure occurs**. Because, when the stress exceeds the yield strength than the plastic deformation occurs which means that the material experiences a permanent deformation.
  - *For brittle materials*, when the **equivalent stress** is greater than **ultimate strength** of the material, than there is going to be a **sudden fracture**.
- Although the stress developed on the component less than yield strength of the material, there can be a **sudden fracture** of the component **if the loading on that component is of dynamic type**. This failure is caused by **"fatigue".**
- The name **"fatigue"** is based on the concept that a material becomes **"tired"** and fails at a stress level below nominal strength of the material.

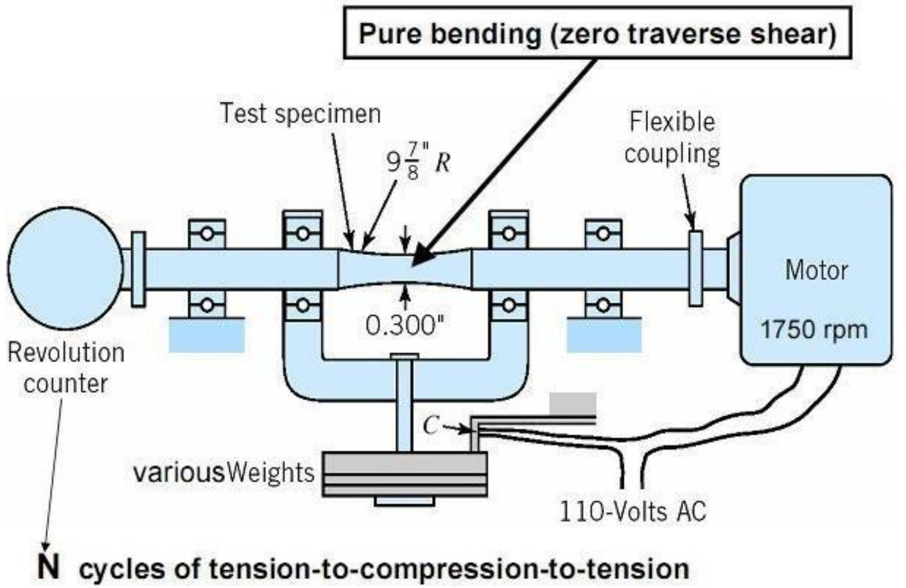
Simple Examples of Fatigue

- You can easily lift 1 kg weight up. However, you may not be successful when you try to lift that weight up and down **a hundred times**. The reason for this unsuccess is that you are **“tired”**.
- If you apply **a bending moment** to a thin wire by your hand, it may not **break at once**, however if you continue to apply **a reversed bending moment repeatedly**, you will notice that the wire is going to be broken **after a while**.

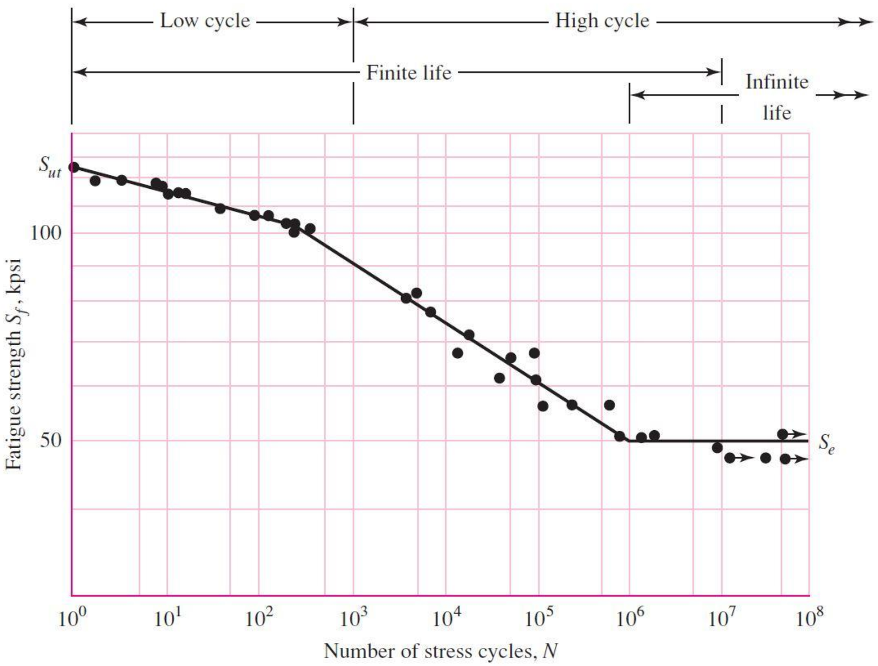
- A fatigue failure has an appearance similar to a brittle fracture, as the fracture surfaces are flat and perpendicular to the stress axis with the absence of necking. However, it is quite different from a static brittle fracture arising from three stages of development.
- **Stage I** is the initiation of one or more **microcracks** due to cyclic plastic deformation on the surface where stress concentrations exist *(step section of shaft, keyway, hole, marks on the surface)*. It is not normally discernible to the naked eye.
- **Stage II** progresses from microcracks to **macrocracks** forming parallel plateau-like fracture surfaces separated by longitudinal ridges.
- **Stage III** occurs during the final stress cycle when the remaining material cannot support the loads, resulting in **a sudden, fast fracture**.



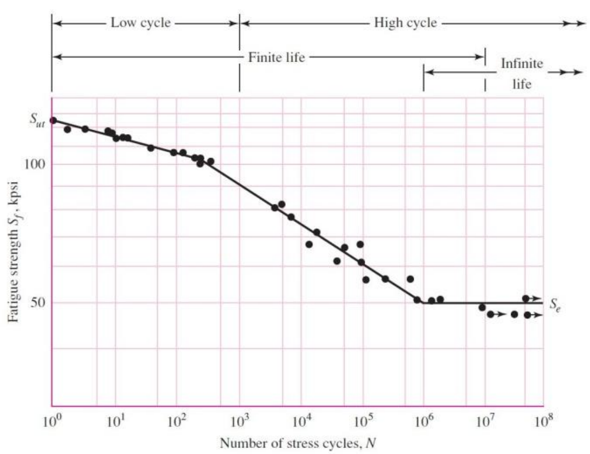
- To determine the strength of materials under the action of fatigue loads, specimens are subjected to repeated or varying forces of specified magnitudes while the **cycles or stress reversals are counted to destruction**.
- The most widely used fatigue-testing device is the high-speed rotating-beam machine. This machine subjects the specimen to **pure bending** *(no transverse shear)* by means of weights. The specimen is very carefully machined and polished.



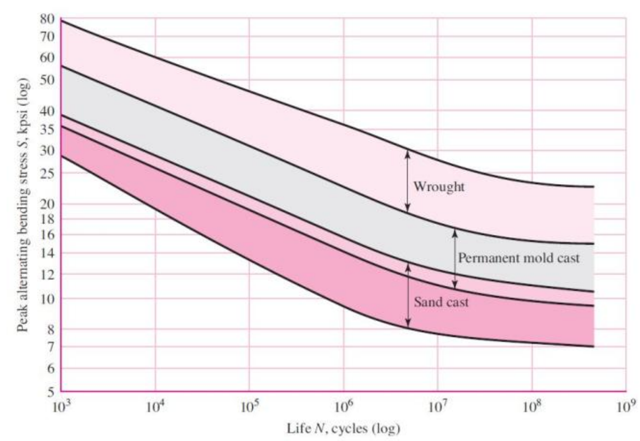
- To establish the fatigue strength of a material, **quite a number of tests are necessary** because of the statistical nature of fatigue.
- For the rotating-beam test, **a constant bending load is applied**, and the number of revolutions of the beam required for failure is recorded.
- The first test is made at a stress that is somewhat under the ultimate strength of the material. The second test is made at a stress that is less than that used in the first. This process is continued, and the results are plotted as an **S-N diagram)**.
- In the case of ferrous metals and alloys, the graph becomes **horizontal** at a certain point (knee point).
- The strength corresponding to this point is named as **"endurance limit".**



- **Endurance limit** is the fatigue strength for an element which has infinite life.
- For ferrous metals and alloys, **infinite life** is assumed as ***(10)^{6}* cycles**.
- For non ferrous materials, there is no knee point but strength value which corresponds to ***(10)^{8}*** or ***5(10)^{8}*** may be considered as the **endurance limit**.
- In the design of many machine components, the elements must be designed as to work for **infinite cycles**. Because fracture of this elements may cause serious problems.



S-N curve for steel



S-N curve for aluminum alloys

- Endurance limit for test specimen ***(S_e')*** is expressed in terms of ultimate strength ***(S_{ut})***.
- **For ductile materials:**

  S_e' =0.5 Sut if S_{ut} < 1400MPa

  S_e' = 700 MPa if S_{ut} ≥ 1400MPa

  For irons:

  S_e' =0.4 Sut if S_{ut} < 400MPa

  S_e' = 160 MPa if S_{ut} ≥ 400MPa

  For aliminums:

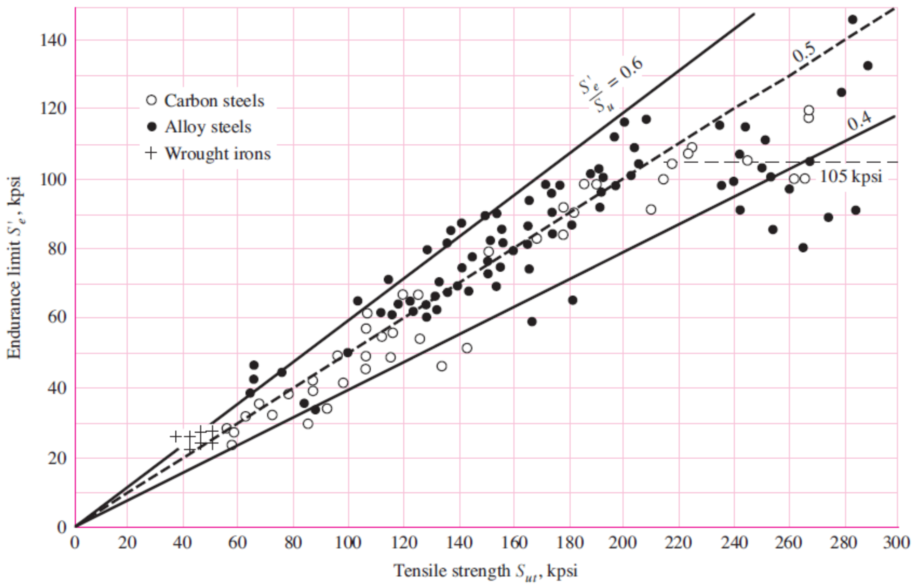
  S_e' =0.4 Sut if S_{ut} < 330MPa

  S_e' = 130 MPa if S_{ut} ≥ 330MPa

  For copper alloys:

  S_e' ≈0.4 Sut if S_{ut} < 280MPa

  S_e' ≈ 100 MPa if S_{ut} ≥ 280MPa



*Endurance limit of the actual machine element is different from these values. **Some factors have detrimental effect** on the life of the machine element.*

$$S_e = k_a k_b k_c k_d k_e S_e'$$

- Machine components are generally designed for infinite life. However, in some special cases, the element may need to be designed for **a finite life**.
- In this case, **fatigue strength** for expected finite life must be calculated from following formula.

$$S_f = 10^C N^b$$

where *C and b are constants*

$$b = -\frac{1}{3}\log\left[\frac{0.8S_u}{S_e}\right] \quad \text{and} \quad c = \log\left[\frac{(0.8S_u)^2}{S_e}\right]$$

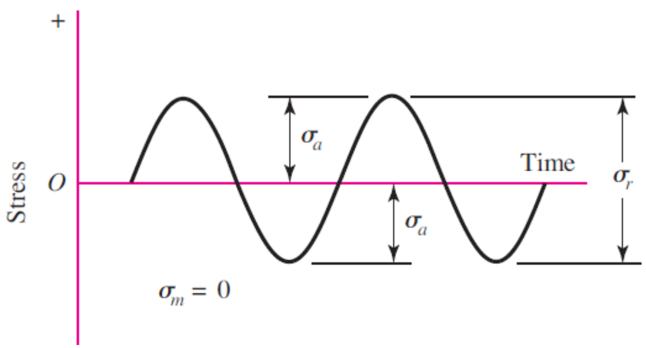
- Fatigue strength ***(S_f)*** will be greater than Endurance limit **(*S_e*)**

Factors Influencing Fatigue Failure

- **Loading:** Axial Tension, Bending, Torsion and Combined
- **Nature and type of loading:** Reversible, Repeated, Fluctuating and Alternating, Mean and Variable components, Frequency of loading and rest periods
- **Geometry:** size effects and stress concentration
- **Material:** composition, structure, directional properties and notch sensitivity
- **Manufacturing:** surface finish, heat treatment and residual stresses Surface residual stress has a significant effect on fatigue life. Compressive residual stresses from machining, cold working, heat treating will oppose a tensile load and thus lower the amplitude of cyclic loading which will improve fatigue life.
- **Environment:** corrosion, high temperature, radiation

Reversed Type

When a rotating machine element is subjected to **a pure bending load**, any element on the surface experiences both tension and compression stresses in one full rotation *(cycle)*.

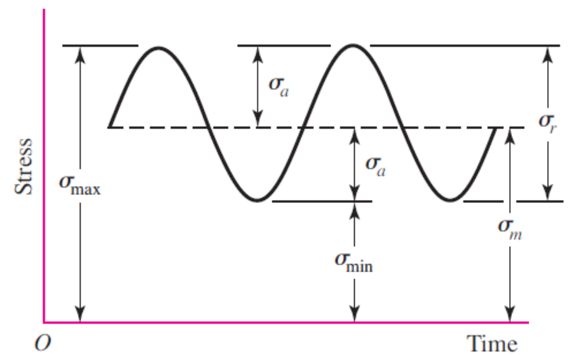


$$\sigma_a = \sigma_{max} = -\sigma_{min}$$

$$\sigma_m = 0$$

Fluctuating Type

In addition to **bending load**, there may be **an axial load** acting on the machine element *(forces developed on helical gear)* then the stress will be fluctuating between two limits where we are going to have alternating and mean components of stresses

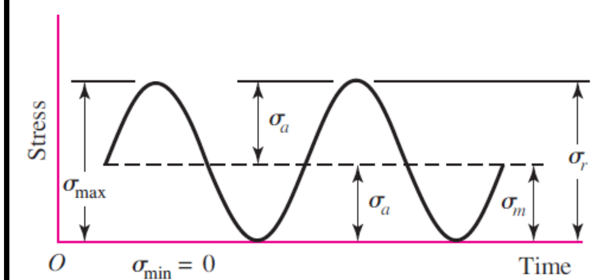


$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Repeated Type

It is a secial case of fluctuating type stress which fluctuating between zero and a maximum value.



$$\sigma_a = \sigma_m = \frac{\sigma_{max}}{2}$$

Surface Factor (k_a)

- The elements may be produced by **forging, hot rolling, cold drawn** and **machining** methods or they may be subjected to some surface finishing operations like **grinding** and **polishing**. Surface effect is calculated with following formula:

$$k_a = aS_{ut}^{\,b}$$

where a and b are constants for different surface finish

ground surface	*a=1.58*	*b=-0.085*
machined	*a=4.51*	*b=-0.265*
cold drawn	*a=4.10*	*b=-0.265*
hot rolled	*a=57.7*	*b=-0.718*
forged	*a=272*	*b=-0.995*

- As it is seen, there is a big difference between **ground** and **forged surfaces**. Thus, surface finish is very important factor which **can not be neglected**.

- **Size factor** is dependent on the size of the element and also on the type of loading.

For bending and torsional loading

For rotating parts having circular cross sections

k_b=1 when $d \leq 8mm$
$k_b = 1.189d^{-0.097}$ when $8 < d \leq 250mm$
k_b=0.6 for larger diameters

For rotating parts having non-circular cross sections, equivalent diameter d_{eq} must be calculated for determination of size factor. In the determination of equivalent diameter, areas at or above of 95% of maximum stress are equated for rotating and non-rotating parts.

For rotating circular cross section $$A_{95} = \frac{\pi}{4}\left[d^2 - \left(0.95d^2\right)\right] = 0.0766d^2$$

For rotating rectangular cross section $$A_{95} = \left[hb - \left(0.95hb\right)\right] = 0.05hb$$

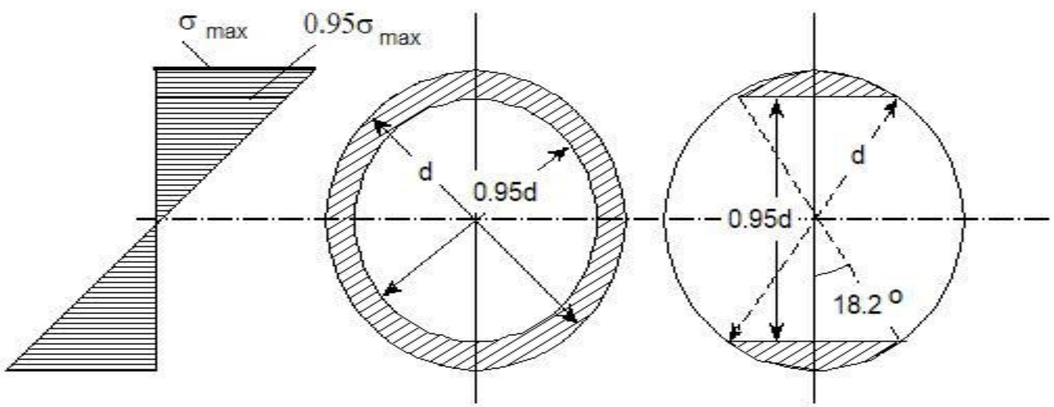
For rotating rectangular cross section $$d_{eq} = \sqrt{\frac{0.05hb}{0.0766}} = 0.808\sqrt{hb}$$

- **Size factor** is dependent on the size of the element and also on the type of loading.

For bending and torsional loading

For non rotating cases, the same approach is used. For circular cross section (non rotating), 95% of maximum stress area is:

$$A_{95} = 4\left[\frac{18.2}{180}\left(\frac{\pi d^2}{4}\right)^2 - \frac{0.156d}{2}\left(\frac{0.95d}{2}\right)\right] = 0.0107d^2$$



Stress distribution

95% of σ max area for rotational part

95% of σ max area for non-rotational part

Equivalent diameter for circular

$$d_{eq} = \sqrt{\frac{0.0107d^2}{0.0766}} = 0.370d$$

Equivalent diameter for rectangular

$$d_{eq} = 0.370\left(0.808\sqrt{hb}\right) = 0.3\sqrt{hb}$$

- **Size factor** is dependent on the size of the element and also on the type of loading.

For axial loading

For pure axial loading

$k_b=1$ and $S_e' = 0.45\ Sut$

For combined loading

Size factor is calculated for bending and then alternating axial stress component is multiplied by axial correction factor "**α**".

$\alpha=1.11$ for $S_{ut} < 1520$ MPa

$\alpha=1.00$ for $S_{ut} > 1520$ MPa

Reliability Factor *(k_c)*

- **Reliability factor** is the result of statistical study on overlapping stress distribution and strength distribution. It may be defined as the probability of not yielding (failure). The following list gives you an idea about reliability factors:

Reliability (R)	***Reliability factor (k_c)***
0,50	*1,000*
0,90	*0,897*
0,95	*0,868*
0,99	*0,814*
0,999	*0,753*
0,999 9	*0,702*
0,999 99	*0,659*
0,999 999	*0,620*
0,999 999 9	*0,584*

Temperature Factor (k_d)

- When operating temperatures are below room temperature, **brittle fracture** is a strong possibility and should be investigated first. When the operating temperatures are higher than room temperature, **yielding** should be investigated first because the **yield strength drops off so rapidly with temperature**.

Temperature, °C	S_T/S_{RT}
20	1.000
50	1.010
100	1.020
150	1.025
200	1.020
250	1.000
300	0.975
350	0.943
400	0.900
450	0.843
500	0.768
550	0.672
600	0.549

where S_T is tensile strength at operating temperature and S_{RT} is tensile strength at room temperature

Assumed as

k_d=1.0 if T < 350 °C

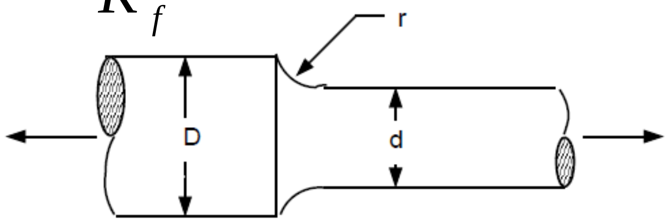
k_d=0.5 if 350 °C < T < 500 °C

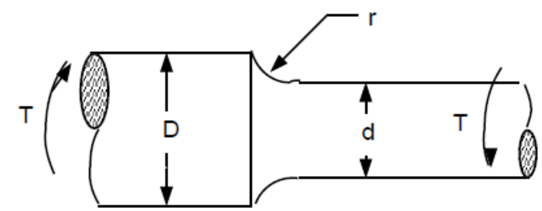
Fatigue Stress Concentration Factor (k_e)

- **Stress concentration factor *(k_e)*** is equal to the reciprocal of the fatigue strength reduction factor which is described as:

$$k_e = \frac{1}{K_f} \quad \text{and} \quad K_f = 1 + q(K_t - 1)$$

where *where **q** is the notch sensitivity factor and **K_t** is the geometric stress concentration factor.*





	D/d =1,02	D/d =1,05	D/d =1,1	D/d=1,5
r/d	Kt	Kt	Kt	Kt
0,025	1,800	-	-	-
0,028	1,728	-	2,200	-
0,031	1,678	2,000	2,125	-
0,037	1,610	1,868	2,020	-
0,044	1,550	1,778	1,938	2,522
0,050	1,508	1,714	1,866	2,400
0,062	1,452	1,626	1,766	2,235
0,075	1,408	1,550	1,684	2,086
0,088	1,370	1,502	1,624	1,970
0,100	1,336	1,457	1,568	1,893
0,125	1,286	1,400	1,496	1,760
0,150	1,254	1,364	1,452	1,662
0,175	1,230	1,340	1,400	1,600
0,200	1,220	1,314	1,372	1,546
0,250	1,216	1,292	1,342	1,508
0,275	1,200	1,270	1,325	1,480
0,300	1,200	1,250	1,296	1,452

	D/d =1,09	D/d =1,20	D/d =1,33	D/d =2,0
r/d	Kt	Kt	Kt	Kt
0,009	-	-	-	-
0,012	1,800	2,300	-	2,600
0,030	1,566	2,040	2,144	2,288
0,025	1,472	1,894	2,020	2,122
0,033	1,384	1,761	1,878	1,966
0,042	1,322	1,644	1,755	1,828
0,050	1,283	1,576	1,677	1,750
0,062	1,244	1,500	1,600	1,644
0,075	1,206	1,434	1,516	1,572
0,087	1,184	1,378	1,458	1,510
0,100	1,166	1,342	1,412	1,466
0,125	1,144	1,275	1,344	1,400
0,150	1,122	1,220	1,294	1,344
0,200	1,110	1,160	1,220	1,266
0,250	1,100	1,130	1,178	1,222
0,300	1,100	1,120	1,160	1,200

Fatigue Stress Concentration Factor *(k_e)*

$$k_e = \frac{1}{K_f} \quad \text{and} \quad K_f = 1 + q(K_t - 1)$$

- If bending stress, shear stress and axial stress have **alternating components**, K_f values for each one of these differ from each other and as a result, **a unique value of k_e** does not exist.
- If this is the case, **use k_e=1** and **increase** various alternating stress components by their respective **strength reduction factors K_f**.
- That is; $K_{fa}\sigma_a \quad K_{fb}\sigma_b \quad K_{ft}\tau_t$
- Increasing the value of stress or decreasing the strength has the same effect on the fatigue life.

- If the loading on the element creates **only alternating stress components**, Von-Mises stress is calculated. For biaxial stress state, **Von-Mises stress is:**

$$\sigma'_a = \sqrt{\sigma_{xa}^{\ 2} + 3\tau_{xya}^{\ 2}}$$

- It is compared with fatigue strength to determine whether the element will fail or not.

For infinite life requirement

$$n = \frac{S_e}{\sigma'_a}$$

For finite life requirement

$$n = \frac{S_f}{\sigma'_a}$$

- If we have **steady (mean) components** of the stresses in addition alternating components, one of the **Fatigue Theories** will be employed in the design or in analyzing the machine elements for fatigue.

- **Modified Goodman** and **Soderberg** approaches are linear theories and commonly used in the design for fatigue strength.
- Soderberg approach which is based on the material yield point, gives **more conservative** results compared to Modified Goodman approach which is based on the material ultimate strength.

Modified Goodman

For infinite life

$$n = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u}}$$

For finite life

$$n = \frac{1}{\frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_u}}$$

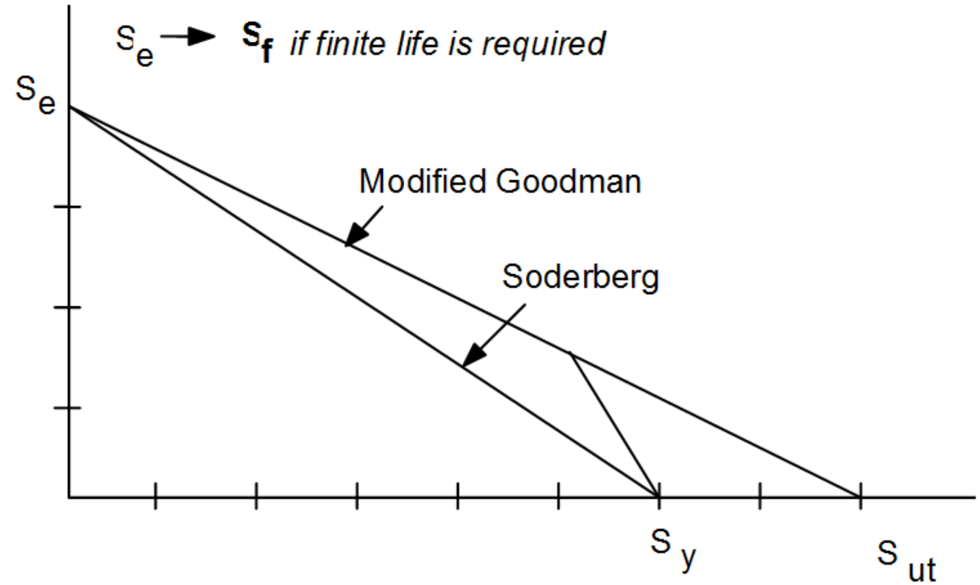
Soderberg

For infinite life

$$n = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y}}$$

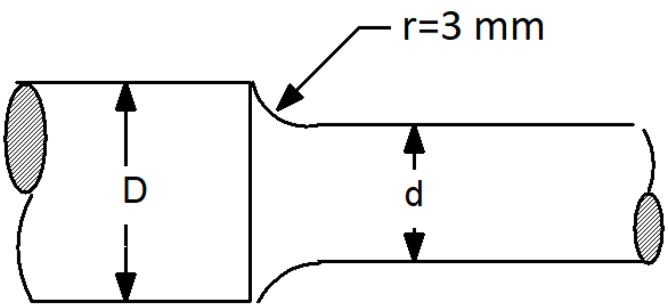
For finite life

$$n = \frac{1}{\frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_y}}$$



TUTORIAL – FATIGUE DESIGN -1

Problem 1: The figure shows a section of an AISI 1050 CD steel shaft with dimensions D=30mm and d=25mm. The shaft section at the shoulder is subjected to a completely reversed bending moment of 60 Nm and a torsion fluctuating between -30 and 50 Nm. Determine the factor of safety for infinite life based on the modified Goodman diagram. (Operating temperature is 100 °C and reliability is %90).



Answer 1:

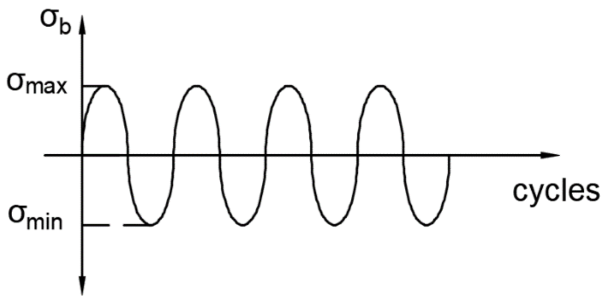
AISI 1050 CD → Sut=690 MPa , Sy=580 MPa (Shigley A-20)

Table A–20

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ($\frac{3}{4}$ to $1\frac{1}{4}$ in). These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.] *Source:* 1986 SAE Handbook, p. 2.15.

1 UNS No.	2 SAE and/or AISI No.	3 Process-ing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197
G10600	1060	HR	680 (98)	370 (54)	12	30	201
G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	25	248

Bending Stress (Reversed)



Maximum and minimum stresses;

$$\sigma_{max} = \frac{Mc}{I} = \frac{32M_{max}}{\pi d^3} = \frac{32.60000}{\pi.25^3} = 39.1\ MPa$$

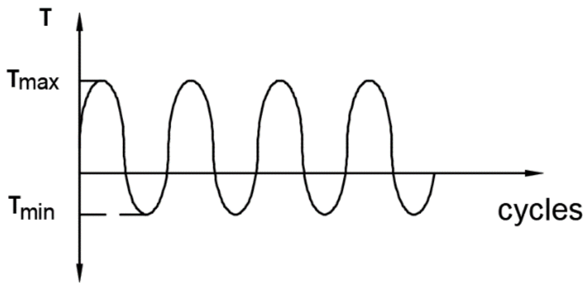
$$\sigma_{min} = -39.1\ MPa$$

Alternating and mean stress components;

$$\sigma_{b_a} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{39.1 - (-39.1)}{2} = 39.1\ MPa$$

$$\sigma_{b_m} = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{39.1 + (-39.1)}{2} = 0\ MPa$$

Torsional Stress (Fluctuating)



Maximum and minimum stresses;

$$\tau_{max} = \frac{Tr}{J} = \frac{16T_{max}}{\pi d^3} = \frac{16.50000}{\pi.25^3} = 16.3\ MPa$$

$$\tau_{min} = \frac{Tr}{J} = \frac{16T_{min}}{\pi d^3} = \frac{16.-30000}{\pi.25^3} = -9.7\ MPa$$

Alternating and mean stress components;

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{16.3 - (-9.7)}{2} = 13\ MPa$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{16.3 + (-9.7)}{2} = 3.3\ MPa$$

Endurance Limit (S_e);

$$S_e = k_a k_b k_c k_d k_e S_e'$$

$$S_e' = 0.5S_{ut} = 0.5.690 = 345\ MPa \quad (S_{ut} < 1400\ MPa)$$

- Surface factor (k_a)

$k_a = aS_{ut}{}^b$ a=4.51, b=-0.265 for machined surface

$$k_a = 4.51(690)^{-0.265} = 0.79$$

Surface Finish	a	b
Ground	1.58	-0.065
Machined or Cold Drawn	4.51	-0.265
Hot Rolled	57.7	-0.718
As Forged	272	-0.995

- Size factor (k_b)

$$k_b = 1.189d^{-0.097} \quad (8 < d < 250\ mm, beam\ is\ rotating)$$

$$k_b = 1.189(25)^{-0.097} = 0.87$$

- Reliability factor (k_c)

$$k_c = 0.897 \quad (for\ 90\%\ reliability, from\ table\ A3 - 19)$$

Table A3-19 Reliability factors kc corresponding to an 8 percent standard deviation of the endurance limit

Reliability R	Standardized variable zr	Reliability factor kc
0,50	0,000	1,000
0,90	1,288	0,897
0,95	1,645	0,868
0,99	2,326	0,814
0,999	3,090	0,753
0,999 9	3,719	0,702
0,999 99	4,265	0,659
0,999 999	4,753	0,620
0,999 999 9	5,199	0,584

- Temperature factor (k_d)

$$k_d = 1.02 \ \ (T = 100\ °C < 350\ °C)$$

Fatigue Failure Resulting from Variable Loading | 291

Table 6–4

Effect of Operating Temperature on the Tensile Strength of Steel.* (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature; $0.099 \leq \hat{\sigma} \leq 0.110$)

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

- Stress concentration factor (k_e)

$$k_e = 1$$

(Stress concentration factor must be taken as 1, because there are two alternating stress components. Therefore, fatigue strength reduction factor values (K_f) for each types of stress will be calculated and multiplied with the corresponding alternating stress components).

- Fatigue strength reduction factors (K_f)

For bending

$q = 0.82$

From table A3-17 based on r&S_{ut}

Table A3-17 Notch-sensitivities for steels and 2024 Wrought Aluminum alloys subjected to reversed bending and reversed axial loads *

		Steels			
Notch rad,(mm)	Aluminum alloys	Sut 0,4GPa	Sut 0,7GPa	Sut 1,0GPa	Sut 1,4GPa
0,000	-	-	-	-	-
0,100	0,200	0,360	0,540	0,670	0,810
0,150	0,250	0,440	0,590	0,710	0,840
0,250	0,300	0,480	0,620	0,740	0,850
0,350	0,380	0,530	0,640	0,760	0,860
0,500	0,410	0,550	0,670	0,790	0,870
0,625	0,450	0,600	0,700	0,810	0,900
0,750	0,490	0,620	0,730	0,830	0,910
0,875	0,520	0,640	0,740	0,840	0,920
1,000	0,540	0,650	0,750	0,850	0,930
1,250	0,590	0,660	0,760	0,860	0,930
1,500	0,630	0,670	0,780	0,870	0,940
2,000	0,680	0,710	0,810	0,890	0,950
2,500	0,730	0,730	0,830	0,900	0,960
4,000	0,830	0,780	0,860	0,930	0,970

For torsion

$q = 0.995$

From table A3-18 for Quenched and drawn steel, r=3mm

Table A3-18 Notch-sensitivities for materials in reversed torsion.

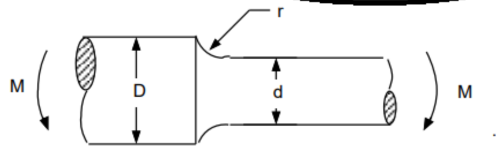
For larger notch radii use the values of q corresponding to r= 4mm.*

Notch radius mm	Quenched and drawn steel q	Annealed steel q	Aluminum alloys q
0,050	0,600	0,400	0,100
0,100	0,800	0,480	0,220
0,250	0,860	0,600	0,370
0,300	0,890	0,670	0,460
0,500	0,915	0,760	0,570
0,750	0,950	0,820	0,670
1,000	0,960	0,860	0,715
1,250	0,970	0,880	0,760
1,500	0,980	0,900	0,790
2,000	0,985	0,930	0,840
2,500	0,990	0,950	0,860
3,000	0,995	0,960	0,890
4,000	0,995	0,990	0,910

K_t=1.5

From table A3-10 based on r/d & D/d

Table A3-10 Stress Concentration factors for round shaft with shoulder fillet in bending



σ_o= Mc/I, where c=d/2 and I= $\pi d^4/64$

	D/d =1,02	D/d =1,05	D/d =1,1	D/d =1,5	D/d =3
r/d	Kt	Kt	Kt	Kt	Kt
0,012	2,290	2,553	2,700	-	-
0,017	2,120	2,378	2,500	3,000	-
0,021	2,000	2,240	2,366	2,774	3,000
0,025	1,926	2,134	2,260	2,600	2,862
0,036	1,760	1,936	2,046	2,310	2,600
0,050	1,644	1,782	1,865	2,060	2,310
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2,140
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1,986
0,087	1,472	1,563	1,630	1,728	1,880
0,100	1,440	1,534	1,580	1,660	1,804
0,125	1,380	1,468	1,500	1,584	1,684
0,150	1,330	1,412	1,450	1,510	1,584

r/d=0.12 D/d=1.2

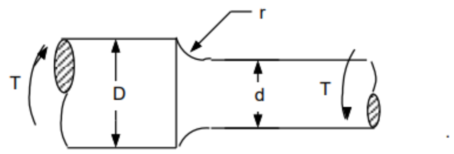
$K_{f_b} = 1 + q(K_t - 1)$

$K_{f_b} = 1 + 0.82(1.5 - 1) = 1.41$

K_t=1.5

(from table A3-9 based on r/d & D/d)

Table A3-9 Stress concentration factors for round shaft with shoulder fillet in torsion



τ_o= Tc/J, where c=d/2 and J=$\pi d^4/32$

r/d=0.12 D/d=1.2

	D/d =1,09	D/d =1,20	D/d =1,33	D/d =2,0
r/d	Kt	Kt	Kt	Kt
0,009	-	-	-	-
0,012	1,800	2,300	-	2,600
0,030	1,566	2,040	2,144	2,288
0,025	1,472	1,894	2,020	2,122
0,033	1,384	1,761	1,878	1,966
0,042	1,322	1,644	1,755	1,828
0,050	1,283	1,576	1,677	1,750
0,062	1,244	1,500	1,600	1,644
0,075	1,206	1,434	1,516	1,572
0,087	1,184	1,378	1,458	1,510
0,100	1,166	1,342	1,412	1,466
0,125	1,144	1,275	1,344	1,400

$K_{f_b} = 1 + q(K_t - 1)$

$K_{f_b} = 1 + 0.995(1.3 - 1) = 1.3$

According to Modified Goodman Theory; (for infinite life, N>10^6 cycles)

$$n = \frac{1}{\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}} \qquad \sigma_a' = \sqrt{\sigma_a{}^2 + 3\tau_a{}^2} \qquad \sigma_m' = \sqrt{\sigma_m{}^2 + 3\tau_m{}^2}$$

Now we need to calculate equivalent stresses (Von Misses Stresses) for both alternating and mean components;

Alternating	Mean
$\sigma_a = \sigma_{b_a}.K_{f_b} = 39.1(1.41) = 55.1\ MPa$	$\sigma_m = \sigma_{b_m} = 0$
$\tau_a = \tau_a.K_{f_t} = 13(1.3) = 16.9\ MPa$	$\tau_m = \tau_m.K_{f_t} = 3.3(1.3) = 4.29\ MPa$
$\sigma_a' = \sqrt{\sigma_a{}^2 + 3\tau_a{}^2} = \sqrt{55.1^2 + 3(16.9)^2}$	$\sigma_m' = \sqrt{\sigma_m{}^2 + 3\tau_m{}^2} = \sqrt{0^2 + 3(4.29)^2}$
$\sigma_a' = 62.4\ MPa$	$\sigma_a' = 7.43\ MPa$

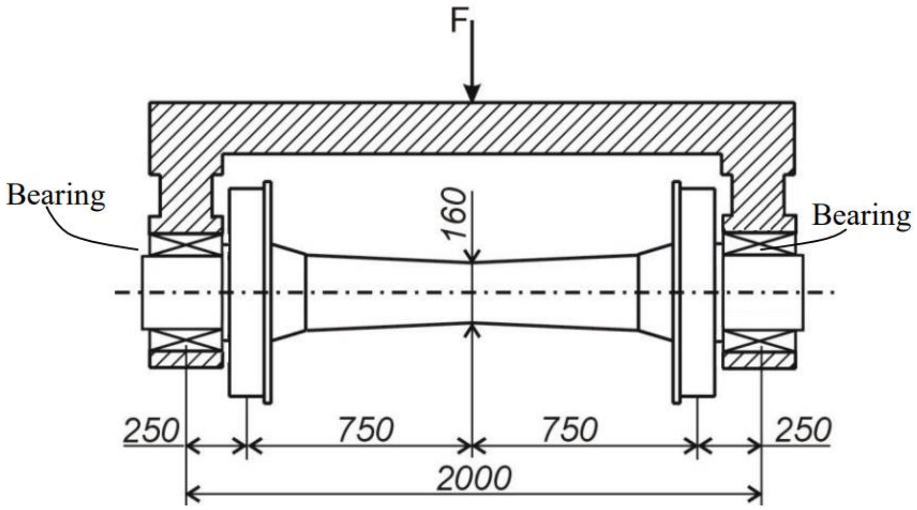
Note: Stress concentration effects both alternating and mean components, because the material is not annealed material, it is quenched material!!!

$S_e = k_a k_b k_c k_d k_e S_e{}'$ =(0.79) (0.87) (0.897) (1.02) (1) (345) =212.7 MPa

$$n = \frac{1}{\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}} = \frac{1}{\frac{62.4}{212.7} + \frac{7.43}{690}} = 3.28 > 1 \qquad safe\ for\ fatigue\ design\ !!!$$

Problem 2: A car axle carries a load of 235 kN. The material is medium carbon forged steel. Its Brinell Hardness number is 179, the ultimate strength is 620 MPa and the yield strength is 500MPa. All dimensions are in mm.

a) Find the value of factor of safety for infinite life at the center of the axle as the car is proceeding along a smooth straight-level track.
b) Estimate the expected life under a completely reversed stress of 200MPa and 400MPa separately.

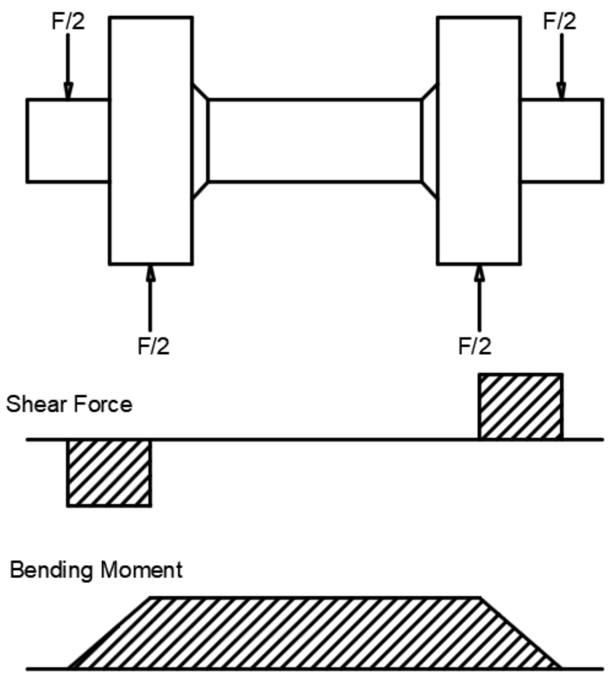


Answer 2:

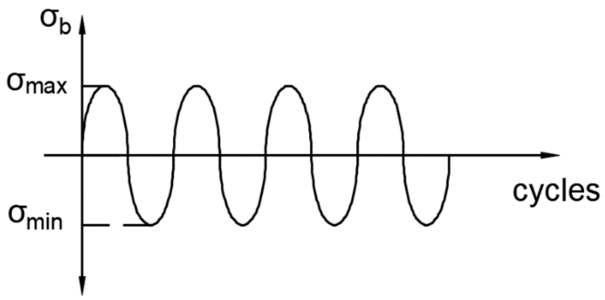
Material= Medium carbon forged steel

HB=179

Sut=620 MPa, Sy=500 MPa



Reversed type of bending stress occurs;



Alternating and mean stresses;

$$\sigma_a = \sigma_{max} = \frac{32M}{\pi d^3} = \frac{32\left(\frac{235}{2}250\right)1000}{\pi 160^3} = 73.1\ MPa$$

$$\sigma_m = 0$$

Von Misses stresses;

$$\sigma_a{}' = \sigma_a = 73.1\ MPa$$

$$\sigma_m{}' = 0$$

Endurance Limit (S_e);

$$S_e = k_a k_b k_c k_d k_e S_e{}'$$

$$S_e{}' = 0.5S_{ut} = 0.5.620 = 310\ MPa \quad (S_{ut} < 1400\ MPa)$$

- Surface factor (k_a)

  $k_a = aS_{ut}{}^b$ a=272, b=-0.995 for forged steel

  $k_a = 272(620)^{-0.995} = 0.453$

Surface Finish	a	b
Ground	1.58	-0.065
Machined or Cold Drawn	4.51	-0.265
Hot Rolled	57.7	-0.718
As Forged	272	-0.995

- Size factor (k_b)
  $k_b = 1.189d^{-0.097} \quad (8 < d < 250mm, beam\ is\ rotating)$
  $k_b = 1.189(160)^{-0.097} = 0.72$
- Reliability factor (k_c)
  $k_c = 1 \quad (nothing\ is\ mentioned)$

- Temperature factor (k_d)
  $k_d = 1 \quad (nothing\ is\ mentioned)$

- Stress concentration factor (k_e)
  $k_e = 1$
  There is no stress concentration at the middle point, such as;
  - Reduction in diameter,
  - Keyway
  - Hole, etc.

$S_e = k_a k_b k_c k_d k_e S_e{}'$

$S_e = (0.453)(0.72)(1)(1)(1)(310) = 101.1\ MPa$

From Modified Goodman Theory;

$$n = \frac{1}{\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}} = \frac{1}{\frac{73.1}{101.1} + 0} = 1.38 > 1 \qquad safe\ for\ fatigue\ design\ !!!$$

ME 307 – Machine Elements I

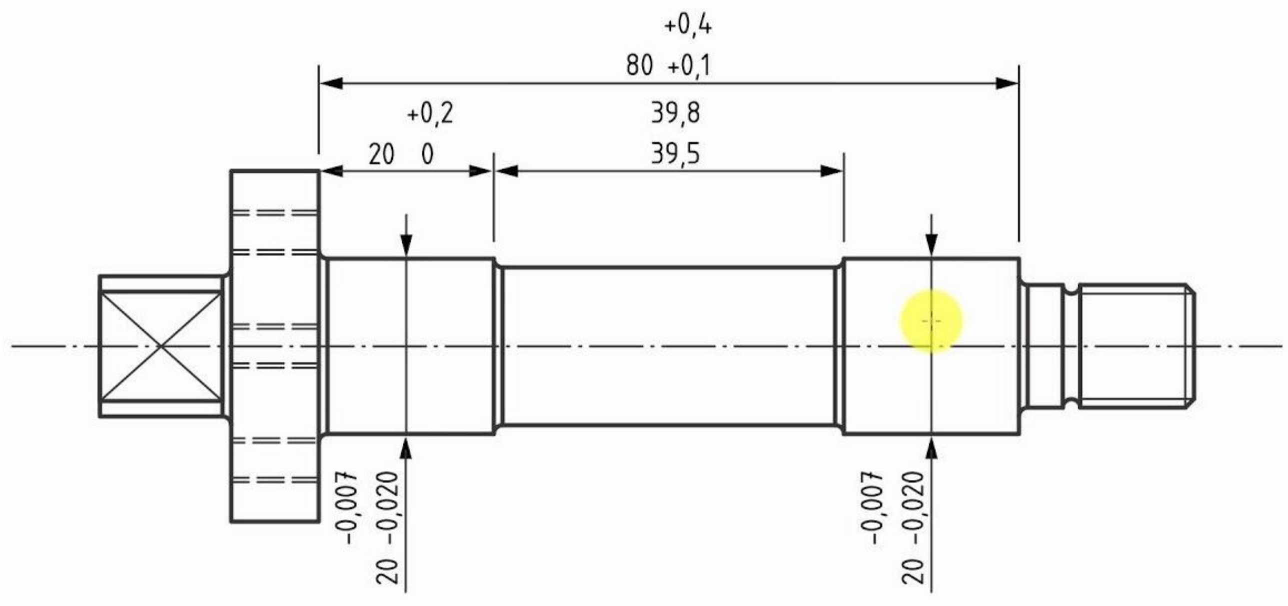
Chapter 6

Tolerances and Fits

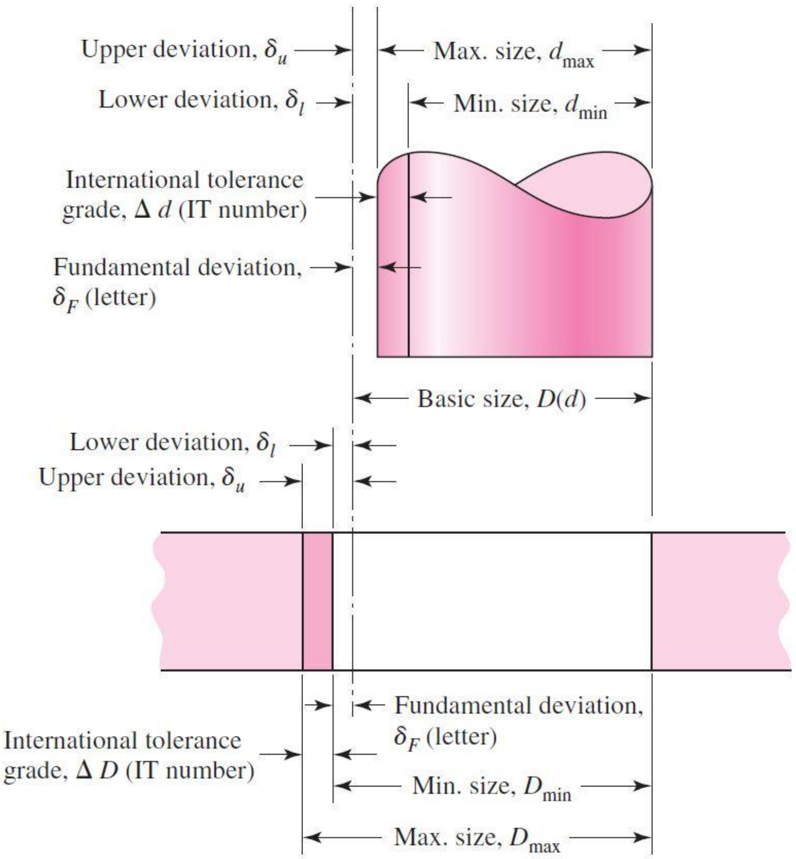
Mechanical Engineering
University of Gaziantep

Dr. Sadık Olguner
Dr. Hakan Çandar

- Designers must keep in mind that, in manufacturing, **it may not be possible** to produce components **at their exact sizes**.
- Production in the **most precise methods** would be **extremely diffucult and costly**.
- However, industry demands that **parts produced shall be between a given maximum and minimum size**.
- The difference between these two sizes is called the **tolerance** which can be defined as **the amount of variation in size which is tolerated**.

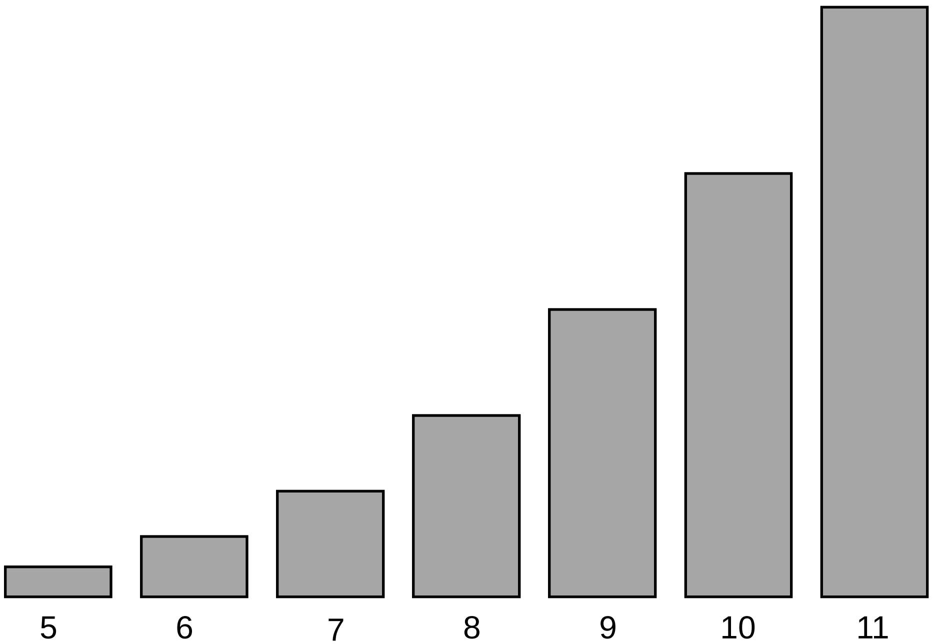


- **Shaft:** a member which fits into another member. It is an element which is fitted into a space between two restrictions.
- **Hole:** a member which houses or fits the shaft. The term hole can apply to the space between two restrictions into which a member has to fit.
- **Basic size:** this is the size about which the limits of a particular fit are fixed. It is the same for both shaft and hole.
- **Deviation:** this is the difference between the actual size and the basic size.
- **Tolerances:** this is the difference between the maximum and minimum limits of size for a hole and shaft. It is also the difference between the upper and lower deviations. Tolerances are designated with a letter and a number **(A9 or h8)**. *The letter indicates the fundamental deviation of tolerance and the number indicates the grade of tolerance*.



Grade of Tolerance

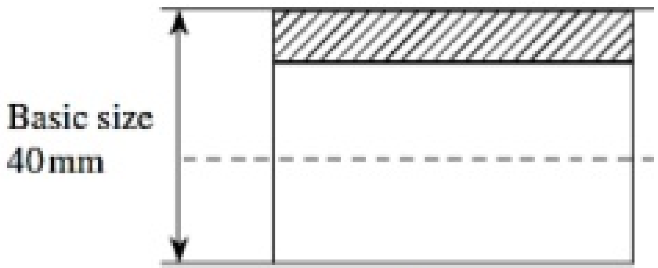
- **Grade of tolerance** represents the size of the tolerance zone and this in turn dictates the degree of accuracy of the machining process required to keep sizes within the specified tolerance. In the ISO system, there are 20 grades of tolerance *(IT qualities which stand for ISO series of tolerances)* ranging from very fine for the lower numbers to extremely coarse for the large numbers *(IT01, IT02, ….., IT14)*.



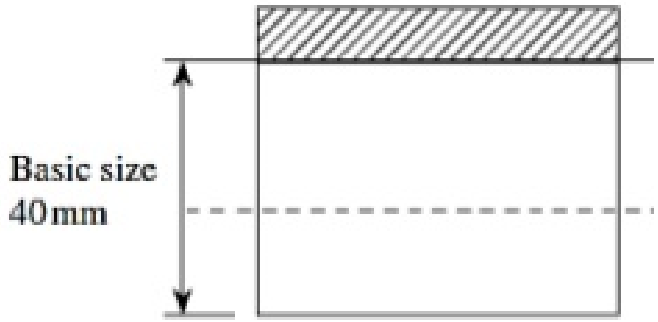
	Nominal Sizes (mm)										
over	1	3	6	10	18	30	50	80	120	180	250
include	3	6	10	18	30	50	80	120	180	250	315
IT Grade											
1	0.8	1	1	1.2	1.5	1.5	2	2.5	3.5	4.5	6
2	1.2	1.5	1.5	2	2.5	2.5	3	4	5	7	8
3	2	2.5	2.5	3	4	4	5	6	8	10	12
4	3	4	4	5	6	7	8	10	12	14	16
5	4	5	6	8	9	11	13	15	18	20	23
6	6	8	9	11	13	16	19	22	25	29	32
7	10	12	15	18	21	25	30	35	40	46	52
8	14	18	22	27	33	39	46	54	63	72	81
9	25	30	36	43	52	62	74	87	100	115	130
10	40	48	58	70	84	100	120	140	160	185	210
11	60	75	90	110	130	160	190	220	250	290	320
12	100	120	150	180	210	250	300	350	400	460	520
13	140	180	220	270	330	390	460	540	630	720	810
14	250	300	360	430	520	620	740	870	1000	1150	1300

Unilateral Limits

This name is given to limits when they are disposed below or above the basic size. *(When the tolerance distribution is only on one side of the basic size)*



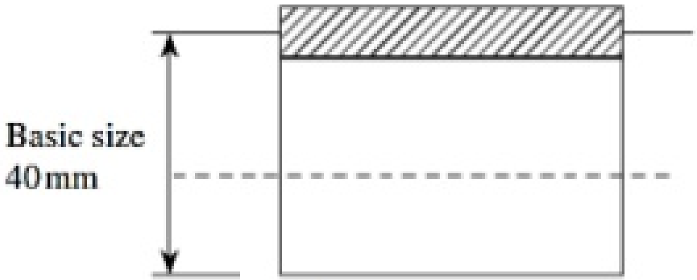
unilateral deviation on negative side



unilateral deviation on positive side

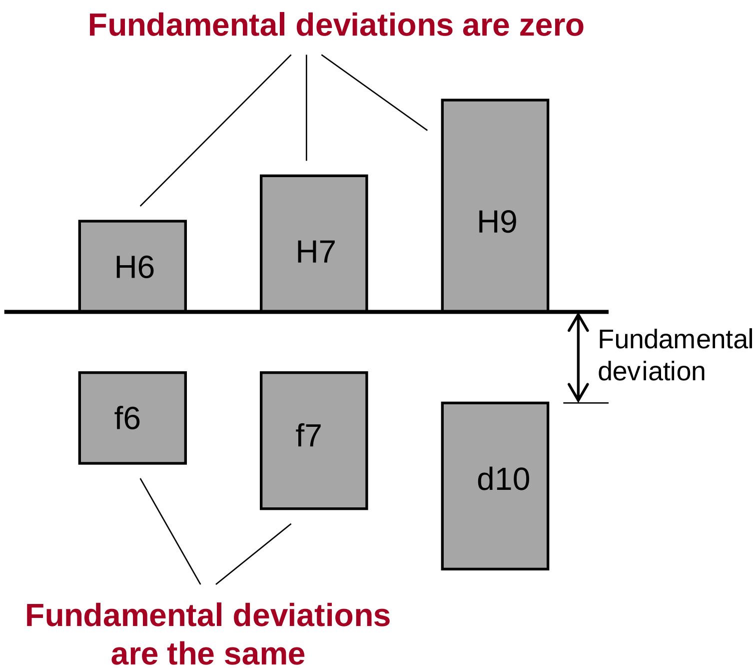
Bilateral Limits

This is the name given to the maximum and minimum limits when they are disposed above and below the basic size respectively. *(When the tolerance distribution lies on either side of the basic size.*



Fundamental Deviation of Tolerance

- **The fundamental deviation** is the deviation which is closest to the basic size and is used to locate the tolerance zone with respect to the basic size. It may be upper or lower deviation depending upon the type of fit and whether the member is a shaft or a hole. *The fundamental deviations are designated by capital letters for holes and lower case letters for shafts.*



In the ISO system, there are 27 positions provided for each grades of tolerance on both shafts and holes.

Fundamental deviations for holes: *A, B, C, CD, D, E, EF, F, FG, G, GH, H, JS, J, K, M, N, P, R, S, T, U, X, Y, Z, ZA, ZB, ZC.*

Fundamental deviations for shafts: *a, b, c, cd, d, ef, f, fg, g, gh, h, js, j, k, m, n, p, r, s, t, u, x, y, z, za, zb, zc.*

Fundamental Deviations for Holes

Nominal Sizes (mm)

over	1	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250
include	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250	280
Grade	All limits below with + sign																			
A	270	270	280	290	290	300	300	310	320	340	360	380	410	460	520	580	660	740	820	920
B	140	140	150	150	150	160	160	170	180	190	200	220	240	260	280	310	340	380	420	480
C	60	70	80	95	95	110	110	120	130	140	150	170	180	200	210	230	240	260	280	300
D	20	30	40	50	50	65	65	80	80	100	100	120	120	145	145	145	170	170	170	190
E	14	20	25	32	32	40	40	50	50	60	60	72	72	85	85	85	100	100	100	110
F	6	10	13	16	16	20	20	25	25	30	30	36	36	43	43	43	50	50	50	56
G	2	4	5	6	6	7	7	9	9	10	10	12	12	14	14	14	15	15	15	17
H	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
J 6	2	5	5	6	6	8	8	10	10	13	13	16	16	18	18	18	22	22	22	25
J 7	4	6	8	10	10	12	12	14	14	18	18	22	22	26	26	26	30	30	30	36
J8	6	10	12	15	15	20	20	24	24	28	28	34	34	41	41	41	47	47	47	55
Js	+/- 0.5T																			
K5	0	0	1	2	2	1	1	2	2	3	3	2	2	3	3	3	2	2	2	3
K6	0	2	2	2	2	2	2	3	3	4	4	4	4	4	4	4	5	5	5	5
K7	0	3	5	6	6	6	6	7	7	9	9	10	10	12	12	12	13	13	13	16
K8	0	5	6	8	8	10	10	12	12	14	14	16	16	20	20	20	22	22	22	25

It will be noticed that *H (hole)* which is featured on the data sheet, is the only one which has the basic size as the lower limit. That fundamental deviation enable a selection of fits to be made on a hole basis.

over	1	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250
include	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250	280
Grade	All limits below with - sign																			
M6	2	1	3	4	4	4	4	4	4	5	5	6	6	8	8	8	8	8	8	9
M7	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M8	-2	+2	+1	+2	+2	+4	+4	+5	+5	+5	+5	+6	+6	+8	+8	+8	+9	+9	+9	+9
N6	4	5	7	9	9	11	11	12	12	14	14	16	16	20	20	20	22	22	22	25
N7	4	4	4	5	5	7	7	8	8	9	9	10	10	12	12	12	14	14	14	14
N8	4	2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5
P6	6	9	12	15	15	18	18	21	21	26	26	30	30	36	36	36	41	41	41	47
R6	10	12	16	20	20	24	24	29	29	35	37	44	47	56	58	61	68	71	75	85
S6	14	16	20	25	25	31	31	38	38	47	53	64	72	85	93	101	113	121	131	149
T6	-	-	-	-	-	-	37	43	49	60	69	84	97	115	127	139	157	171	187	209
U6	18	20	25	30	30	37	44	55	65	81	96	117	137	163	183	203	227	249	275	306
V6	-	-	-	-	36	43	51	63	76	96	114	139	165	195	221	245	275	301	331	376
X6	20	25	31	37	42	50	60	75	92	116	140	171	203	241	273	303	341	376	416	466
Y6	-	-	-	-	-	59	71	89	109	138	168	207	247	293	333	373	416	461	511	571
Z6	26	32	39	47	57	69	84	107	131	166	204	251	303	358	408	458	511	566	631	701
P7	6	8	9	11	11	14	14	17	17	21	21	24	24	28	28	28	33	33	33	36
R7	10	11	13	16	16	20	20	25	25	30	32	38	41	48	50	53	60	63	67	74
S7	14	15	17	21	21	27	27	34	34	42	48	58	66	77	85	93	105	113	123	138
T7	-	-	-	-	-	-	33	39	45	55	64	78	91	107	119	131	149	163	179	198
U7	18	19	22	26	26	33	40	51	61	76	91	111	131	155	175	195	219	241	267	295
V7	-	-	-	-	32	39	47	59	72	91	109	133	159	187	213	237	267	293	323	365
X7	20	24	28	33	38	46	56	71	88	111	135	165	197	233	265	295	333	368	408	455
P8	6	12	15	18	18	22	22	26	26	32	32	37	37	43	43	43	50	50	50	56
R8	10	15	19	23	23	28	28	34	34	41	43	51	54	63	65	68	77	80	84	94
S8	14	19	23	28	28	35	35	43	43	53	59	71	79	92	100	108	122	130	140	158
T8	-	-	-	-	-	-	41	48	54	66	75	91	104	122	134	146	166	180	196	218
U8	18	23	28	33	33	41	48	60	70	87	102	124	144	170	190	210	236	258	284	315
V8	-	-	-	-	39	47	55	68	81	102	120	146	172	202	228	252	284	310	340	385

Fundamental Deviations for Shafts

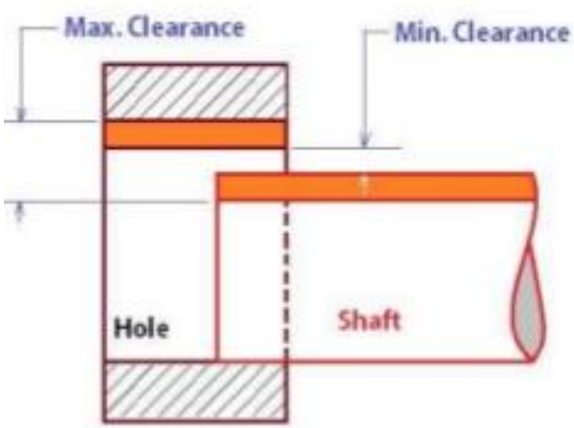
	Nominal Sizes (mm)																			
over	1	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250
include	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250	280
Grade	All limits below with – sign																			
a	270	270	280	290	290	300	300	310	320	340	360	380	410	460	520	580	660	740	820	920
b	140	140	150	150	150	160	160	170	180	190	200	220	240	260	280	310	340	380	420	480
c	60	70	80	95	95	110	110	120	130	140	150	170	180	200	210	230	240	260	280	300
d	20	30	40	50	50	65	65	80	80	100	100	120	120	145	145	145	170	170	170	190
e	14	20	25	32	32	40	40	50	50	60	60	72	72	85	85	85	100	100	100	110
f	6	10	13	16	16	20	20	25	25	30	30	36	36	43	43	43	50	50	50	56
g	2	4	5	6	6	7	7	9	9	10	10	12	12	14	14	14	15	15	15	17
h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j(5&6)	2	2	2	3	3	4	4	5	5	7	7	9	9	11	11	11	13	13	13	16
j7	4	4	5	6	6	8	8	10	10	12	12	15	15	18	18	18	21	21	21	26
js	+/-0.5T																			
Grade	All limits below with + sign																			
k (4 to 7)	0	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	4	4	4	4
k from 8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
m	2	4	6	7	7	8	8	9	9	11	11	13	13	15	15	15	17	17	17	20
n	4	8	10	12	12	15	15	17	17	20	20	23	23	27	27	27	31	31	31	34
p	6	12	15	18	18	22	22	26	26	32	32	37	37	43	43	43	50	50	50	56
r	10	15	19	23	23	28	28	34	34	41	43	51	54	63	65	68	77	80	84	94
s	14	19	23	28	28	35	35	43	43	53	59	71	79	92	100	108	122	130	140	158
t	-	-	-	-	-	-	41	48	54	66	75	91	104	122	134	146	166	180	196	218
u	18	23	28	33	33	41	48	60	70	87	102	124	144	170	190	210	236	258	284	315
v	-	-	-	-	39	47	55	68	81	102	120	146	172	202	228	252	284	310	340	385
x	20	28	34	40	45	54	64	80	97	122	146	178	210	248	280	310	350	385	425	475
y	-	-	-	-	-	63	75	94	114	144	174	214	254	300	340	380	425	470	520	580
z	26	35	42	50	60	73	88	112	136	172	210	258	310	365	415	465	520	575	640	710

It will be noticed that *h (shaft)* has basic size at the upper limit. That fundamental deviation enable a selection of fits to be made on a shaft basis.

Fit may be defined as the relative motion which can exist between a shaft and a hole resulting from their final sizes. There are three classes of fit in common use.

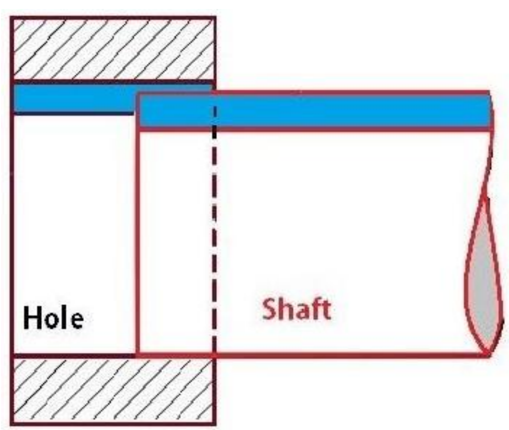
Clearance Fit

This fit results when the shaft size is always less than the hole size for all possible combinations within their tolerance ranges. **The minimum clearance** occurs at the maximum shaft size and the minimum hole size. **Maximum clearance** occurs at the minimum shaft size and the maximum hole size. ***($d_u < D_L$)***



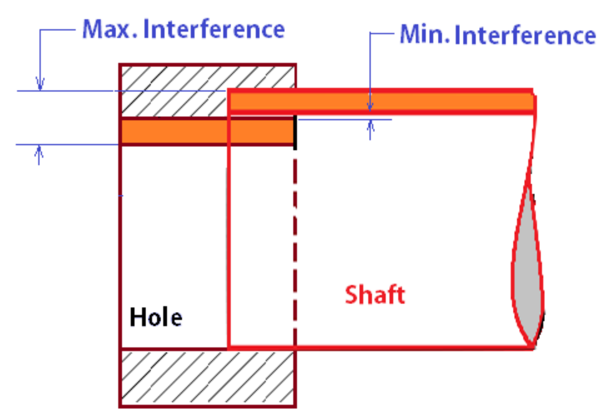
Transition Fit

This fit results when the tolerances are such that the largest hole is greater than the smallest shaft and the largest shaft is greater than the smallest hole. Tolerance zones of the shaft and the hole are overlapped. ***($d_u > D_L$ & $D_u > d_L$)***

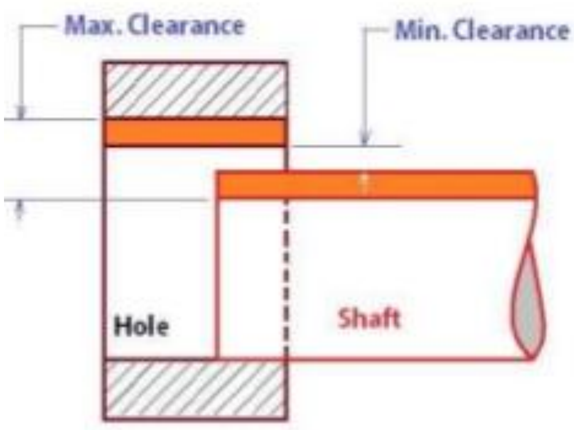


Interference Fit

This fit results when the smallest shaft is greater than the largest hole. **The minimum interference** occurs at the minimum shaft size and the maximum hole size. **The maximum interference** occurs at the maximum shaft size and the minimum hole size. ***($d_L > D_u$)***



Clearance Fit



Shaft

$30^{-0.01}_{-0.03}$ — upper deviation (d_u), lower deviation (d_L)

$29.97 - 29.99$ (lower limit – upper limit) $T_s = d_u - d_L$

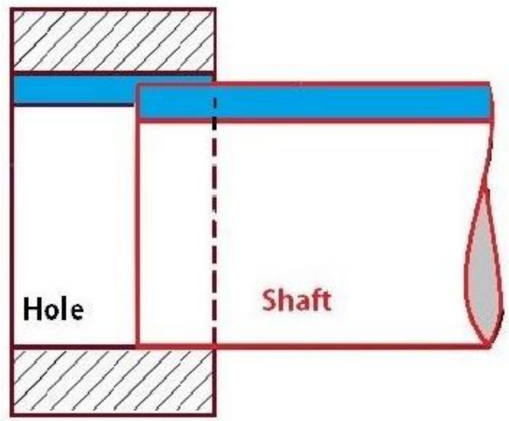
Hole

$30^{0.05}_{0}$ — upper deviation (D_u), lower deviation (D_L)

$30.00 - 30.05$ (lower limit – upper limit) $T_H = D_u - D_L$

($d_u < D_L$)
$-0.01 < 0$

Transition Fit



Shaft

$30^{+0.01}_{-0.02}$ — upper deviation (d_u), lower deviation (d_L)

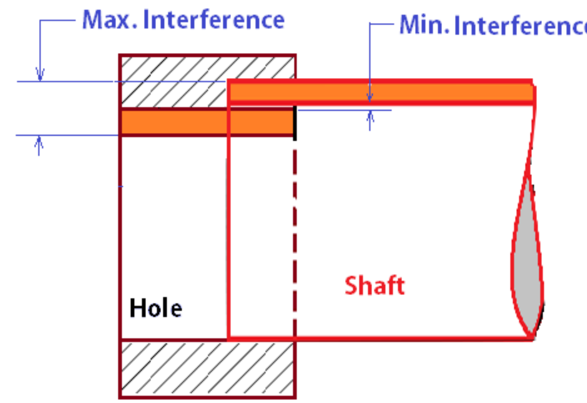
$29.98 - 30.01$ (lower limit – upper limit) $T_s = d_u - d_L$

Hole

$30^{0.02}_{0}$ — upper deviation (D_u), lower deviation (D_L)

$30.00 - 30.02$ (lower limit – upper limit) $T_H = D_u - D_L$

Interference Fit



Shaft

$30^{0.05}_{0.03}$ — upper deviation (d_u), lower deviation (d_L)

$30.03 - 30.05$ (lower limit – upper limit) $T_s = d_u - d_L$

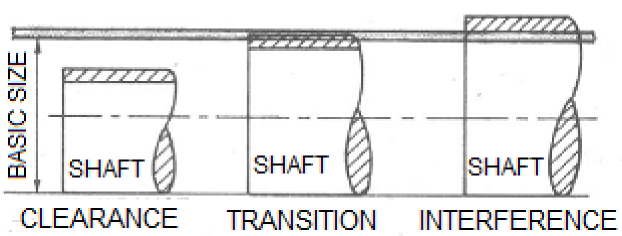
Hole

$30^{0.02}_{0}$ — upper deviation (D_u), lower deviation (D_L)

$30.00 - 30.02$ (lower limit – upper limit) $T_H = D_u - D_L$

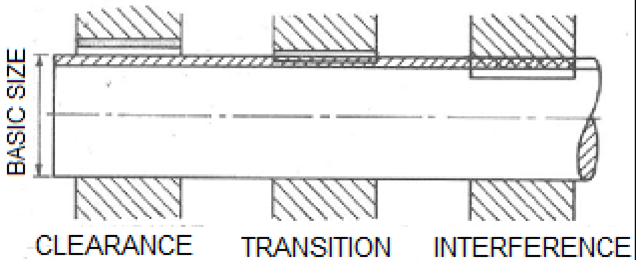
($d_L > D_u$)
$0.03 > 0.02$

- Depending upon having either hole or shaft fundamental deviation zero, fits are named as to be based on either basic hole or basic shaft system.
- **Basic Hole System:** Fits are obtained by regarding the hole as standard with a zero fundamental deviation and varying the fundamental deviations of the shaft to suit.



The basic hole system is most commonly used because it is easier to produce standart holes by drilling or reaming and the turning the shaft to suit the fit desired. This is especially true when the elements are produced at their standard sizes *(antififiriction bearings etc.)*. Measurements can also be made more quickly on shaft sizes than on hole sizes.

- **Basic Shaft System:** In this case, fundamental deviation of shaft is zero and the fits are obtained by varying the fundamental deviations of the holes.



In some cases, there may be more than one transmission element *(gear, pulley etc.)* to be mounted on a shaft and the fit may require different ranges of tolerances for different parts of the shaft. Due to the difficulties of performing different machining operation on different sections of the shaft, it may be better to have bores of mounting elements *(gear etc.)* to be machined to suit the shaft. Tolerances of holes may be adjusted to have required fit.

Example

Consider a fit which is designated as **H9/d10.** If the basic size is 100 mm, determine the values of the tolerance limits for hole and shaft.

This is the fit based on **Basic Hole System (BHS).**

Nominal Sizes (mm)

over	1	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250
include	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250	280
Grade	All limits below with + sign																			
A	270	270	280	290	290	300	300	310	320	340	360	380	410	460	520	580	660	740	820	920
B	140	140	150	150	150	160	160	170	180	190	200	220	240	260	280	310	340	380	420	480
C	60	70	80	95	95	110	110	120	130	140	150	170	180	200	210	230	240	260	280	300
D	20	30	40	50	50	65	65	80	80	100	100	120	120	145	145	145	170	170	170	190
E	14	20	25	32	32	40	40	50	50	60	60	72	72	85	85	85	100	100	100	110
F	6	10	13	16	16	20	20	25	25	30	30	36	36	43	43	43	50	50	50	56
G	2	4	5	6	6	7	7	9	9	10	10	12	12	14	14	14	15	15	15	17
H	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
J 6	2	5	5	6	6	8	8	10	10	13	13	16	16	18	18	18	22	22	22	25
J 7	4	6	8	10	10	12	12	14	14	18	18	22	22	26	26	26	30	30	30	36
J8	6	10	12	15	15	20	20	24	24	28	28	34	34	41	41	41	47	47	47	55
Js	+/- 0.5T																			
K5	0	0	1	2	2	1	1	2	2	3	3	2	2	3	3	3	2	2	2	3
K6	0	2	2	2	2	2	2	3	3	4	4	4	4	4	4	4	5	5	5	5
K7	0	3	5	6	6	6	6	7	7	9	9	10	10	12	12	12	13	13	13	16
K8	0	5	6	8	8	10	10	12	12	14	14	16	16	20	20	20	22	22	22	25

Nominal Sizes (mm)

over	1	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250
inc.	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250	280
Grade	All limits below with – sign																			
a	270	270	280	290	290	300	300	310	320	340	360	380	410	460	520	580	660	740	820	920
b	140	140	150	150	150	160	160	170	180	190	200	220	240	260	280	310	340	380	420	480
c	60	70	80	95	95	110	110	120	130	140	150	170	180	200	210	230	240	260	280	300
d	20	30	40	50	50	65	65	80	80	100	100	120	120	145	145	145	170	170	170	190
e	14	20	25	32	32	40	40	50	50	60	60	72	72	85	85	85	100	100	100	110
f	6	10	13	16	16	20	20	25	25	30	30	36	36	43	43	43	50	50	50	56
g	2	4	5	6	6	7	7	9	9	10	10	12	12	14	14	14	15	15	15	17
h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
j(5&6)	2	2	2	3	3	4	4	5	5	7	7	9	9	11	11	11	13	13	13	16
j7	4	4	5	6	6	8	8	10	10	12	12	15	15	18	18	18	21	21	21	26

Nominal Sizes (mm)

over	1	3	6	10	18	30	50	80	120	180	250
inc.	3	6	10	18	30	50	80	120	180	250	315
IT											
1	0.8	1	1	1.2	1.5	1.5	2	2.5	3.5	4.5	6
2	1.2	1.5	1.5	2	2.5	2.5	3	4	5	7	8
3	2	2.5	2.5	3	4	4	5	6	8	10	12
4	3	4	4	5	6	7	8	10	12	14	16
5	4	5	6	8	9	11	13	15	18	20	23
6	6	8	9	11	13	16	19	22	25	29	32
7	10	12	15	18	21	25	30	35	40	46	52
8	14	18	22	27	33	39	46	54	63	72	81
9	25	30	36	43	52	62	74	87	100	115	130
10	40	48	58	70	84	100	120	140	160	185	210
11	60	75	90	110	130	160	190	220	250	290	320
12	100	120	150	180	210	250	300	350	400	460	520
13	140	180	220	270	330	390	460	540	630	720	810
14	250	300	360	430	520	620	740	870	1000	1150	1300

Hole: $100\left\{\dfrac{87}{0}\right\}$

Shaft: $100\left\{\dfrac{-120}{-260}\right\}$

For Basic Shaft System, for h; *A, B, C, D, E, F, and G fits*, in the calculation of tolerances, **IT quality is to be added to determine upper tolerance value.**

For Basic Shaft System, for h; *J, K, M, N, P, R, S, T, U, X, Z, ZB, and ZC fits*, in the calculation of tolerances, **IT quality is to be subtracted to determine lower tolerance value.**

Nominal Sizes (mm)																				
over	1	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250
inc.	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250	280
Grade	All limits below with + sign																			
A	270	270	280	290	290	300	300	310	320	340	360	380	410	460	520	580	660	740	820	920
B	140	140	150	150	150	160	160	170	180	190	200	220	240	260	280	310	340	380	420	480
C	60	70	80	95	95	110	110	120	130	140	150	170	180	200	210	230	240	260	280	300
D	20	30	40	50	50	65	65	80	80	100	100	120	120	145	145	145	170	170	170	190
E	14	20	25	32	32	40	40	50	50	60	60	72	72	85	85	85	100	100	100	110
F	6	10	13	16	16	20	20	25	25	30	30	36	36	43	43	43	50	50	50	56
G	2	4	5	6	6	7	7	9	9	10	10	12	12	14	14	14	15	15	15	17
H	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
J 6	2	5	5	6	6	8	8	10	10	13	13	16	16	18	18	18	22	22	22	25
J 7	4	6	8	10	10	12	12	14	14	18	18	22	22	26	26	26	30	30	30	36
J8	6	10	12	15	15	20	20	24	24	28	28	34	34	41	41	41	47	47	47	55
Js	+/- 0.5T																			
K5	0	0	1	2	2	1	1	2	2	3	3	2	2	3	3	3	2	2	2	3
K6	0	2	2	2	2	2	2	3	3	4	4	4	4	4	4	4	5	5	5	5
K7	0	3	5	6	6	6	6	7	7	9	9	10	10	12	12	12	13	13	13	16
K8	0	5	6	8	8	10	10	12	12	14	14	16	16	20	20	20	22	22	22	25
Grade	All limits below with - sign																			
M6	2	1	3	4	4	4	4	4	4	5	5	6	6	8	8	8	8	8	8	9
M7	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M8	-2	+2	+1	+2	+2	+4	+4	+5	+5	+5	+5	+6	+6	+8	+8	+8	+9	+9	+9	+9
N6	4	5	7	9	9	11	11	12	12	14	14	16	16	20	20	20	22	22	22	25
Z6	26	32	39	47	57	69	84	107	131	166	204	251	303	358	408	458	511	566	631	701
P7	6	8	9	11	11	14	14	17	17	21	21	24	24	28	28	28	33	33	33	36
R7	10	11	13	16	16	20	20	25	25	30	32	38	41	48	50	53	60	63	67	74
S7	14	15	17	21	21	27	27	34	34	42	48	58	66	77	85	93	105	113	123	138
over	1	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250
inc.	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250	280

Nominal Sizes (mm)																				
over	1	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250
inc.	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250	280
Grade	All limits below with – sign																			
e	14	20	25	32	32	40	40	50	50	60	60	72	72	85	85	85	100	100	100	110
f	6	10	13	16	16	20	20	25	25	30	30	36	36	43	43	43	50	50	50	56
g	2	4	5	6	6	7	7	9	9	10	10	12	12	14	14	14	15	15	15	17
h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
j(5&6)	2	2	2	3	3	4	4	5	5	7	7	9	9	11	11	11	13	13	13	16
j7	4	4	5	6	6	8	8	10	10	12	12	15	15	18	18	18	21	21	21	26

	Nominal Sizes (mm)										
over	1	3	6	10	18	30	50	80	120	180	250
inc.	3	6	10	18	30	50	80	120	180	250	315
IT											
6	6	8	9	11	13	16	19	22	25	29	32
7	10	12	15	18	21	25	30	35	40	46	52
8	14	18	22	27	33	39	46	54	63	72	81
9	25	30	36	43	52	62	74	87	100	115	130
10	40	48	58	70	84	100	120	140	160	185	210

$$h6/J7 \rightarrow 40\left\{\frac{0}{-16}\right\}/\left\{\frac{14}{-11}\right\} \rightarrow transition\_fit$$

$$h6/R7 \rightarrow 40\left\{\frac{0}{-16}\right\}/\left\{\frac{-25}{-50}\right\} \rightarrow interference\_fit$$

$$h6/C8 \rightarrow 40\left\{\frac{0}{-16}\right\}/\left\{\frac{159}{120}\right\} \rightarrow clearance\_fit$$

Examples for BHS

For Basic Hole System, for H; *a, b, c, d, e, f, and g fits*, in the calculation of tolerances, **IT quality is to be subtracted to determine lower tolerance value.**

For Basic Hole System, for H; *j, k, m, n, p, r, s, t, u, x, z, zb, and zc fits*, in the calculation of tolerances, **IT quality is to be added to determine upper tolerance value.**

	Nominal Sizes (mm)																			
over	1	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250
inc.	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250	280
Grade	All limits below with + sign																			
E	14	20	25	32	32	40	40	50	50	60	60	72	72	85	85	85	100	100	100	110
F	6	10	13	16	16	20	20	25	25	30	30	36	36	43	43	43	50	50	50	56
G	2	4	5	6	6	7	7	9	9	10	10	12	12	14	14	14	15	15	15	17
H	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
J 6	2	5	5	6	6	8	8	10	10	13	13	16	16	18	18	18	22	22	22	25
J 7	4	6	8	10	10	12	12	14	14	18	18	22	22	26	26	26	30	30	30	36

	Nominal Sizes (mm)																			
over	1	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250
inc.	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250	280
Grade	All limits below with – sign																			
a	270	270	280	290	290	300	300	310	320	340	360	380	410	460	520	580	660	740	820	920
b	140	140	150	150	150	160	160	170	180	190	200	220	240	260	280	310	340	380	420	480
c	60	70	80	95	95	110	110	120	130	140	150	170	180	200	210	230	240	260	280	300
d	20	30	40	50	50	65	65	80	80	100	100	120	120	145	145	145	170	170	170	190
e	14	20	25	32	32	40	40	50	50	60	60	72	72	85	85	85	100	100	100	110
f	6	10	13	16	16	20	20	25	25	30	30	36	36	43	43	43	50	50	50	56
g	2	4	5	6	6	7	7	9	9	10	10	12	12	14	14	14	15	15	15	17
h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
j(5&6)	2	2	2	3	3	4	4	5	5	7	7	9	9	11	11	11	13	13	13	16
j7	4	4	5	6	6	8	8	10	10	12	12	15	15	18	18	18	21	21	21	26
js										+/-0.5T										
Grade	All limits below with + sign																			
n	4	8	10	12	12	15	15	17	17	20	20	23	23	27	27	27	31	31	31	34
p	6	12	15	18	18	22	22	26	26	32	32	37	37	43	43	43	50	50	50	56
r	10	15	19	23	23	28	28	34	34	41	43	51	54	63	65	68	77	80	84	94
s	14	19	23	28	28	35	35	43	43	53	59	71	79	92	100	108	122	130	140	158

$$H6/j5 \rightarrow 40\begin{Bmatrix}16\\ \hline 0\end{Bmatrix}/\begin{Bmatrix}6\\ \hline -5\end{Bmatrix} \rightarrow transition\_fit$$

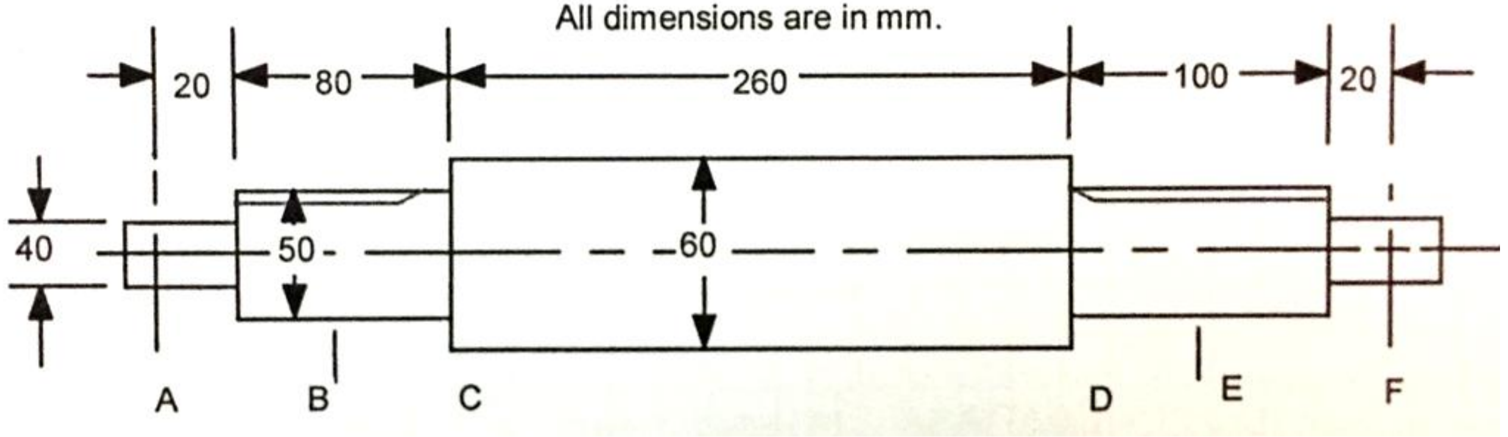
$$H6/r5 \rightarrow 40\begin{Bmatrix}16\\ \hline 0\end{Bmatrix}/\begin{Bmatrix}45\\ \hline 34\end{Bmatrix} \rightarrow interference\_fit$$

$$H6/e7 \rightarrow 40\begin{Bmatrix}16\\ \hline 0\end{Bmatrix}/\begin{Bmatrix}-50\\ \hline -75\end{Bmatrix} \rightarrow clearance\_fit$$

	Nominal Sizes (mm)										
over	1	3	6	10	18	30	50	80	120	180	250
İnc.	3	6	10	18	30	50	80	120	180	250	315
IT											
4	3	4	4	5	6	7	8	10	12	14	16
5	4	5	6	8	9	11	13	15	18	20	23
6	6	8	9	11	13	16	19	22	25	29	32
7	10	12	15	18	21	25	30	35	40	46	52
8	14	18	22	27	33	39	46	54	63	72	81

Example

The shaft shown in figure below has radial bearings at A and F. The tolerance on the bore diameter of the bearings is **{ 0 ; 0.012 mm }**. If the fit is of **transition type**, and the maximum tolerance on fit **(T_F) is 0.028 mm**, determine tolerance range of the shaft and give its ISO designation.

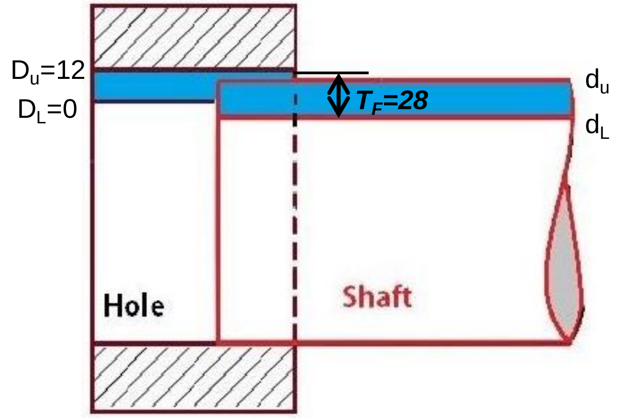


Solution

$$Hole \Rightarrow 40\begin{pmatrix} 0.012 \\ 0 \end{pmatrix} \Rightarrow BHS$$

	Nominal Sizes (mm)										
over	1	3	6	10	18	30	50	80	120	180	250
inc.	3	6	10	18	30	50	80	120	180	250	315
IT											
4	3	4	4	5	6	7	8	10	12	14	16
5	4	5	6	8	9	11	13	15	18	20	23
6	6	8	9	11	13	16	19	22	25	29	32
7	10	12	15	18	21	25	30	35	40	46	52

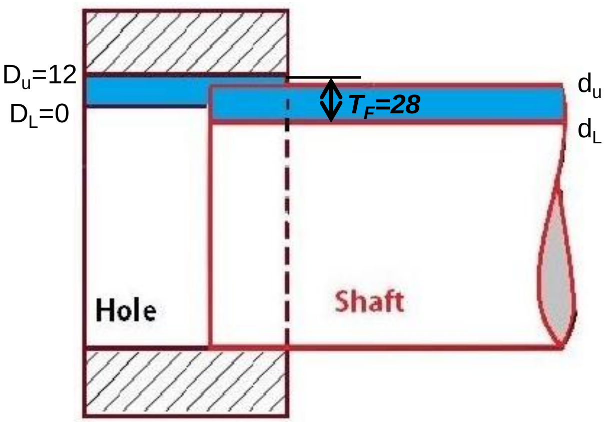
$$Hole \Rightarrow H5\begin{pmatrix} 0.011 \\ 0 \end{pmatrix}$$



$d_L > D_U - T_F = 12 - 28 = -16$

d_L>-16 and **d_u>0**

Example



$d_L > D_U - T_F = 12 - 28 = -16$

$\mathbf{d_L > -16}$ **and** $\mathbf{d_u > 0}$

	Nominal Sizes (mm)										
over	1	3	6	10	18	30	50	80	120	180	250
inc.	3	6	10	18	30	50	80	120	180	250	315
IT											
4	3	4	4	5	6	7	8	10	12	14	16
5	4	5	6	8	9	11	13	15	18	20	23
6	6	8	9	11	13	16	19	22	25	29	32
7	10	12	15	18	21	25	30	35	40	46	52

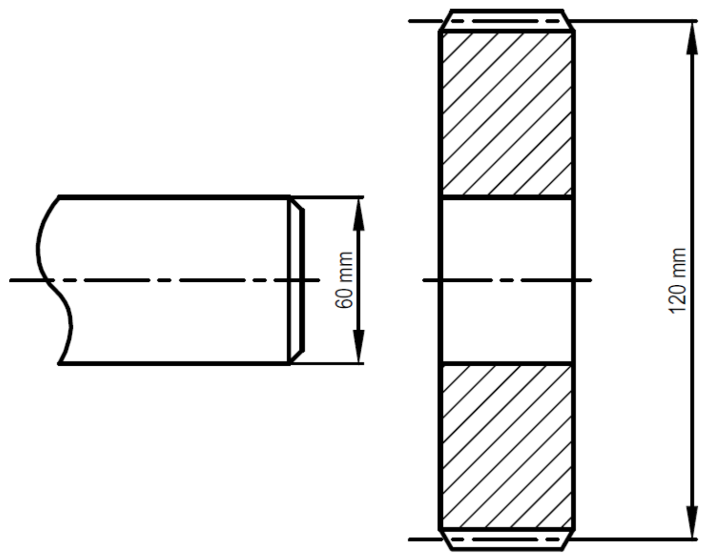
$$j7\begin{pmatrix}+15\\-10\end{pmatrix} \quad \text{or} \quad h6\begin{pmatrix}0\\-16\end{pmatrix}$$

$H5/j7$ or $H5/h6$ can be used.

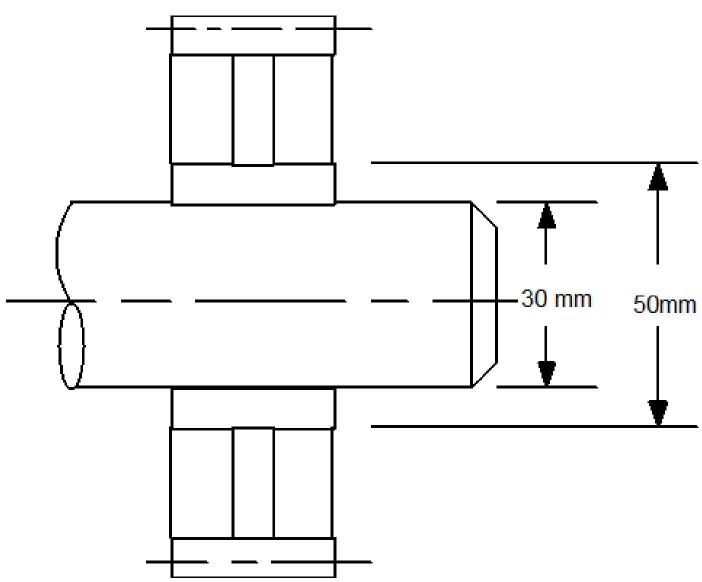
	Nominal Sizes (mm)																			
over	1	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250
include	3	6	10	14	18	24	30	40	50	65	80	100	120	140	160	180	200	225	250	280
Grade	All limits below with – sign																			
a	270	270	280	290	290	300	300	310	320	340	360	380	410	460	520	580	660	740	820	920
b	140	140	150	150	150	160	160	170	180	190	200	220	240	260	280	310	340	380	420	480
c	60	70	80	95	95	110	110	120	130	140	150	170	180	200	210	230	240	260	280	300
d	20	30	40	50	50	65	65	80	80	100	100	120	120	145	145	145	170	170	170	190
e	14	20	25	32	32	40	40	50	50	60	60	72	72	85	85	85	100	100	100	110
f	6	10	13	16	16	20	20	25	25	30	30	36	36	43	43	43	50	50	50	56
g	2	4	5	6	6	7	7	9	9	10	10	12	12	14	14	14	15	15	15	17
h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j(5&6)	2	2	2	3	3	4	4	5	5	7	7	9	9	11	11	11	13	13	13	16
j7	4	4	5	6	6	8	8	10	10	12	12	15	15	18	18	18	21	21	21	26
js										+/-0.5T										
Grade	All limits below with + sign																			
k (4 to 7)	0	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	4	4	4	4
k from 8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
m	2	4	6	7	7	8	8	9	9	11	11	13	13	15	15	15	17	17	17	20
n	4	8	10	12	12	15	15	17	17	20	20	23	23	27	27	27	31	31	31	34
p	6	12	15	18	18	22	22	26	26	32	32	37	37	43	43	43	50	50	50	56
r	10	15	19	23	23	28	28	34	34	41	43	51	54	63	65	68	77	80	84	94
s	14	19	23	28	28	35	35	43	43	53	59	71	79	92	100	108	122	130	140	158
t	-	-	-	-	-	-	41	48	54	66	75	91	104	122	134	146	166	180	196	218
u	18	23	28	33	33	41	48	60	70	87	102	124	144	170	190	210	236	258	284	315
v	-	-	-	-	39	47	55	68	81	102	120	146	172	202	228	252	284	310	340	385
x	20	28	34	40	45	54	64	80	97	122	146	178	210	248	280	310	350	385	425	475
y	-	-	-	-	-	63	75	94	114	144	174	214	254	300	340	380	425	470	520	580
z	26	35	42	50	60	73	88	112	136	172	210	258	310	365	415	465	520	575	640	710

TUTORIAL-Tolerances and Fits

Problem 1: A gear made from steel is to be assembled on a steel shaft with an interference fit. Nominal diameter of the shaft is 60mm and the pitch diameter of the gear is 120mm. The shaft and the gear hole are machined with IT5 and IT6 qualities respectively. If the fit is based on basic shaft system (BSS), determine the nearest standard fit if the minimum interference is 0.013 mm.



Problem 2: A shaft of 30 mm is machined to IT5 quality. A gear produced from cast iron is to be press-fitted over the shaft as shown in the figure below. The outside diameter of the hub of the gear is 50 mm. The minimum torque to be transmitted by the gear shaft assembly is 25 Nm. The maximum tangential stress in the gear hub is required to be 82 Mpa. Modulus of elasticities and poisson's ratios for cast iron and for steel are E_h=100GPa and E_s=207GPa; ϑ_h=0.211 ϑ_s=0,3 respectively. Determine the nearest standard fit using Basic Shaft System (BSS) if coefficient of friction, f=0.1 and Length of the fit is 20mm.



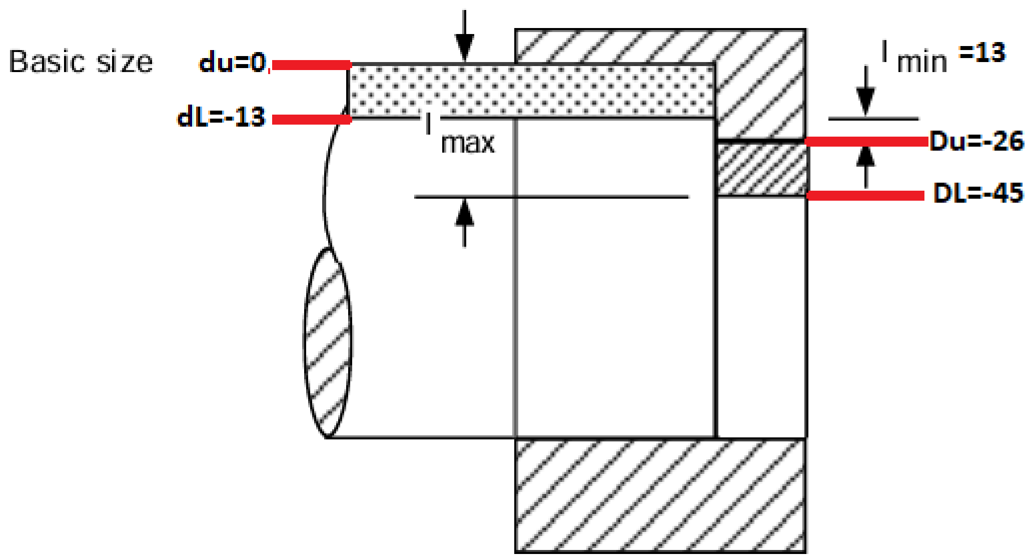
ANSWERS

Answer 1

Fitting is based on BSS, therefore fundamental deviation for shaft will be zero,
Shaft=> IT5 => h5 => $60\left(\frac{0}{-13}\right) - 13$ (from table A4-2, IT quality is taken as 13 for IT grade 5 and for 60mm diameter.)

Iso standard of the hole will be determined, IT quality is given as 6,
Hole=> IT6 => ?

I_{min}=0,013 mm = 13µm



Hole=> $60\left(\frac{-26}{-45}\right) - 19$ => from table A4-3, nearest standard is selected as P6.

Result= h5/P6

Answer 2

Nominal dia. =30mm
BSS => h5
IT quality =5
Pressed fit (interference fit)
Hub dia=50 mm
T_{min}=25Nm
$\sigma_{t\ max}$=82 MPa
f=0,1 (coefficient of friction)
L=20 mm (fit length)

$A = 2.\pi.r.L$ $\qquad$ $A = 2.\pi.15.20 = 1885\ mm^2$

$T_{min} = p_{min}.A.f.\frac{d}{2}$ $\qquad$ $25000 = p_{min}.(1885).(0,1).(15)$ $\qquad$ $p_{min} = 8.84\ MPa$

Lame Equations:

$\sigma_{t_{max}} = p_{max}.\frac{c^2+b^2}{c^2-b^2}$ $\qquad$ $82 = p_{max}.\frac{25^2+15^2}{25^2-15^2}$ $\qquad$ $p_{max} = 38,6\ MPa$

Radial Interference:

$\delta = \delta_{hole} + \delta_{shaft}$

$\delta = \frac{bp}{E_h}\left\{\frac{c^2+b^2}{c^2-b^2} + \mu_h\right\} + \frac{bp}{E_s}\left\{\frac{b^2+a^2}{b^2-a^2} - \mu_s\right\}$ shaft is solid, a=0 , materials are different;

Then the equation is reduced to;

$$\delta = \frac{bp}{E_h}\left\{\frac{c^2+b^2}{c^2-b^2} + \mu_h\right\} + \frac{bp}{E_s}\{1 - \mu_s\}$$

Hole (gear)
- Eh=100 GPa (cast iron)
- μh=0,211 (poisson's ratio)
- b=15mm , c=25mm

Shaft
- Es=207 GPa (steel)
- μs=0,3 (poisson's ratio)
- a=0 , b=15mm

$\delta_{min} = \frac{15.(8,84)}{100000}\left\{\frac{25^2+15^2}{25^2-15^2} + 0,211\right\} + \frac{15.(8,84)}{207000}\{1 - 0,3\}$ $\qquad$ $\delta_{min} = 0,0035mm$

$\delta_{max} = \delta_{min}\left(\frac{p_{max}}{p_{min}}\right) = 0,0035\frac{38,6}{8,84} = 0,015mm$

Diamentral interference:

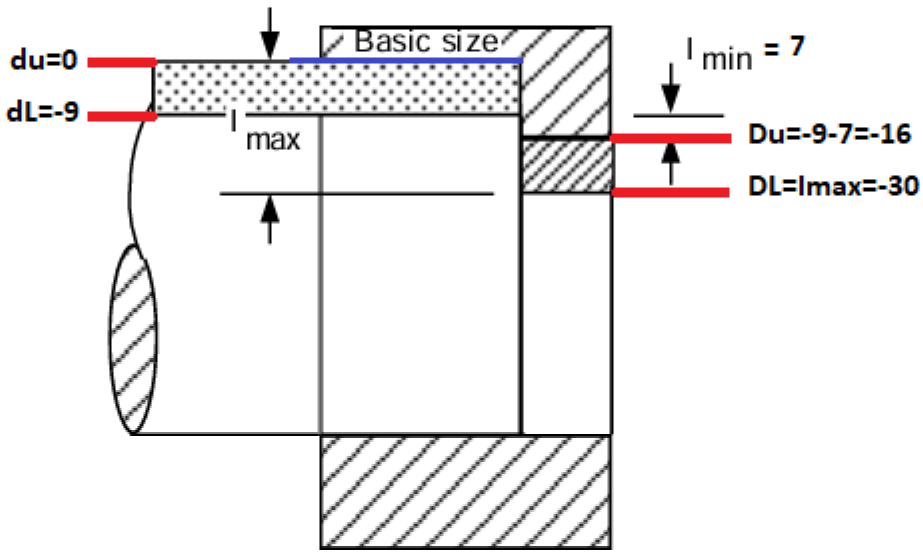
$$I_{min} = 2.\delta_{min} = 2.(0{,}0035) = 0{,}007mm = 7\mu m$$

$$I_{max} = 2.\delta_{max} = 2.(0{,}015) = 0{,}030mm = 30\mu m$$

Interference fit for BSS

Shaft;
BSS => h5 => $30\left(\frac{0}{-9}\right) - 9$
d_U=0 µm
d_L=-9 µm

I_{min}=7 µm
I_{max}=30 µm



From the figure;
D_U=-9-7=-16 µm
D_L=I_{max}=-30 µm

So, the tolerance limits of the hole is $30\left(\frac{-16}{-30}\right)$

Nearest standard between this values can be selected;

Selection procedures:

1- $D_U \leq -16\ \ (I_{min} = 7)\quad and\ \ D_L \geq -30\ \ (I_{max} = 30)$

2- Since DU and DL are negative, we are looking the grades under K8 and for upper deviation. P6, which is -18 near to -16. It is ok.

3- When the IT grade is then added; (-18-13)=-31. -31<-30 but it is ok.

Result = h5/P6 = $\left(\frac{0}{-9}\right) / \left(\frac{-18}{-31}\right)$

Upper and lower limits;

Hole = 29,969mm – 29,982mm

Shaft = 29,910mm – 30,000mm

DESIGN OF POWER SCREWS

<u>Power Screw:</u> A power screw is a device used in machinery to change angular motion into recti-linear motion and usually, to transmit power.

Usign Areas:

- Lead screws of machine tools
- Screw for presses and jacks
  - friction drive forging press
  - linear jack
  - screw jack
  - clamps
- Machine tool drivers
- Steering system in automobiles

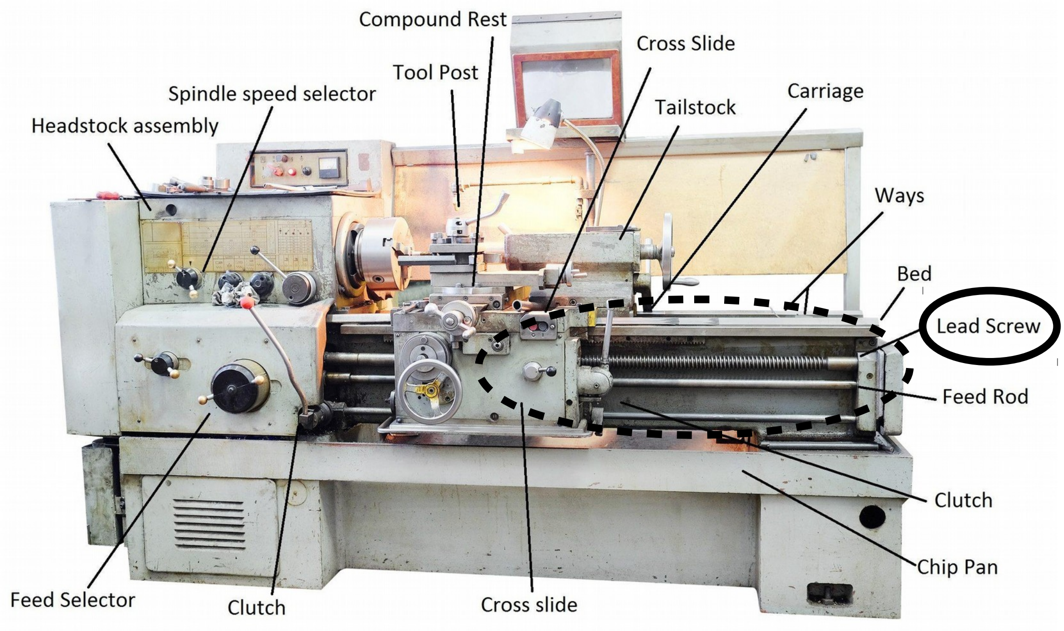


Figure 1 Lathe machine (lead screw)

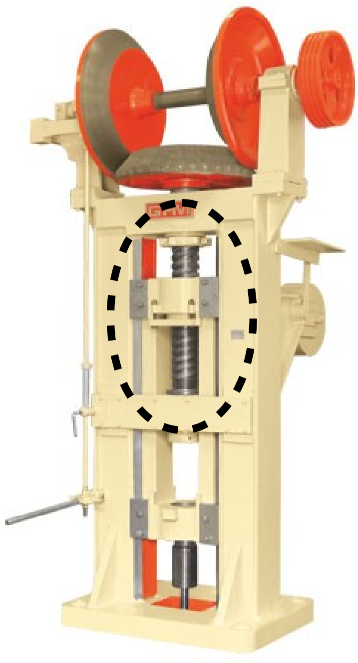


Figure 2: Friction drive forging press

Figure 3: Screw clamp

Figure 4: Screw jack

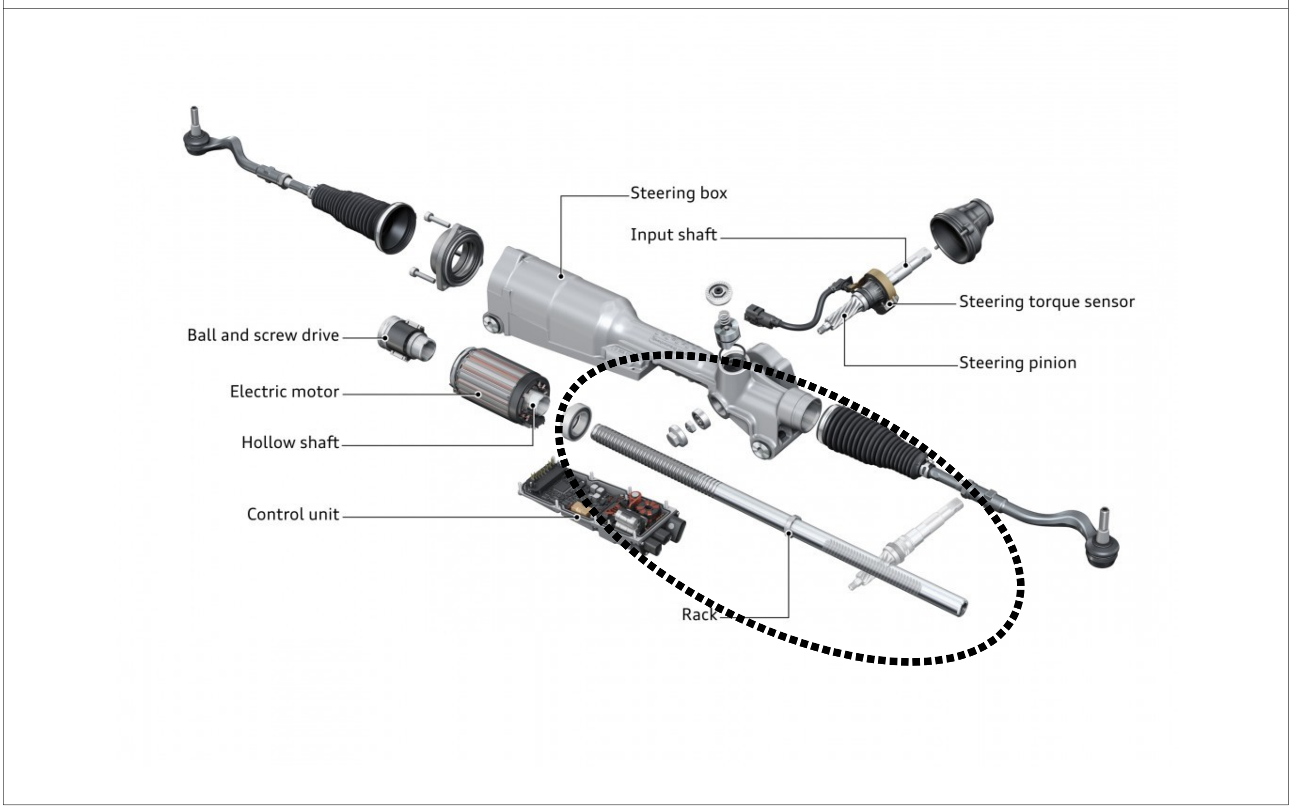


Figure 5: Car steering

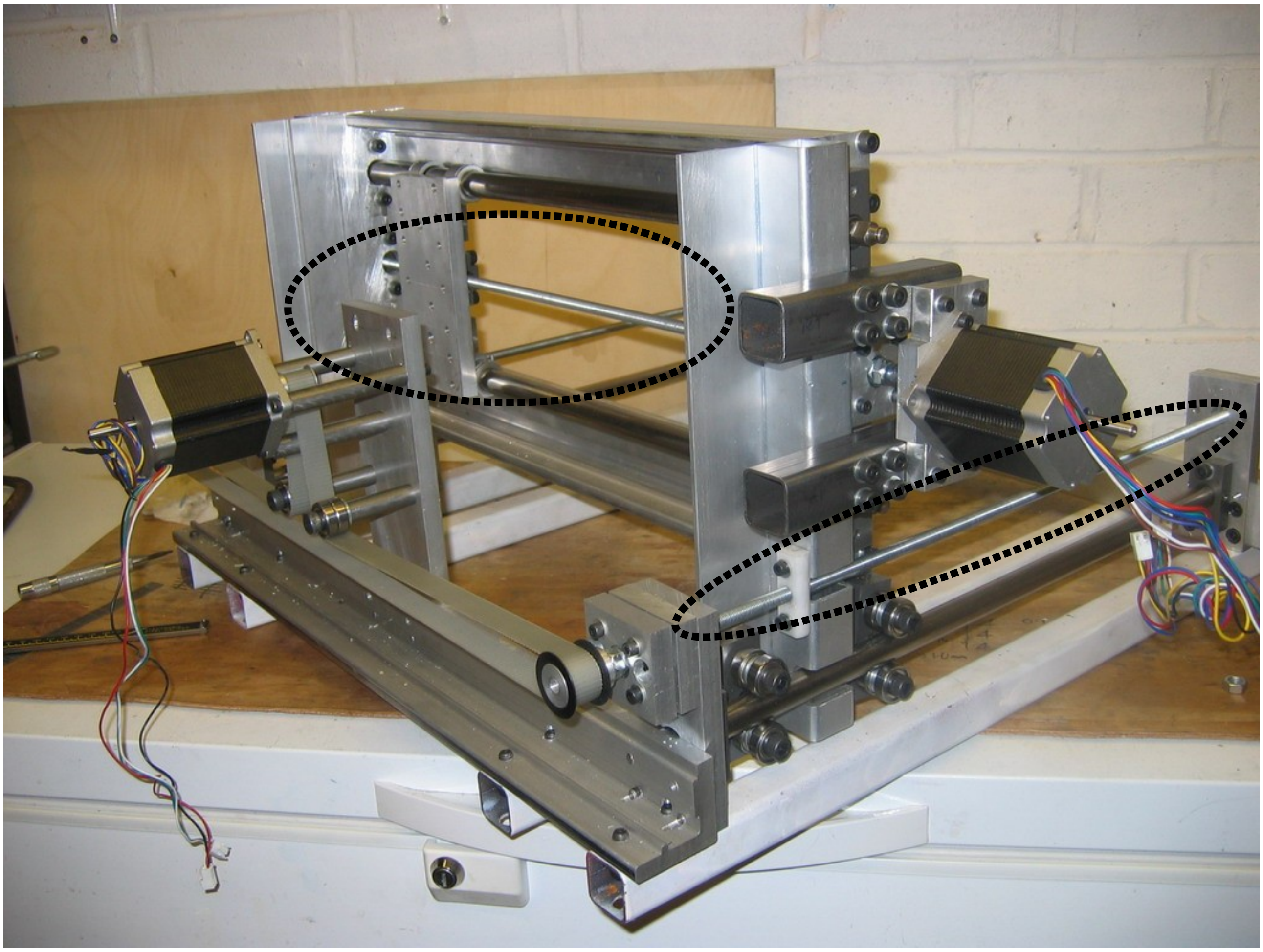


Figure 6: Usage of power screw as machine tool driver in CNC machines

Typical Threaded Element;

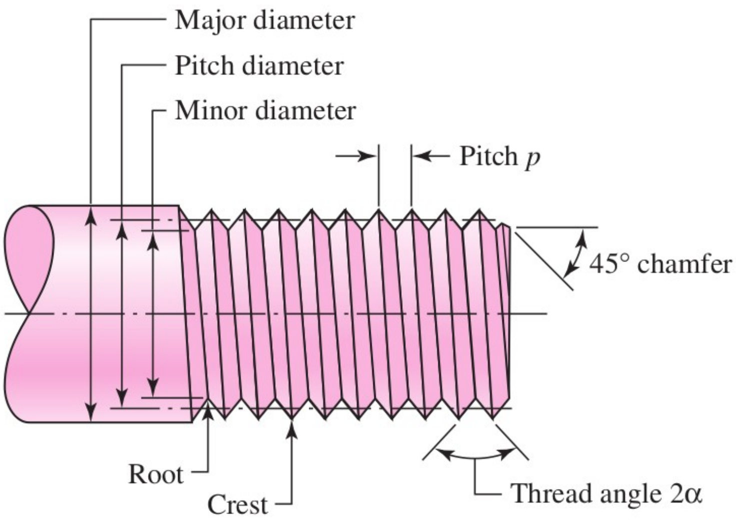


Figure 7: A typical threaded element

d: Major diameter
d_m: Mean (pitch) diameter $d_m = d - p/2$
dr: Minor (root) diameter $d_r = d - p$

Diameter for tensile stress area (d_t) : This is the diameter for the determination of tensile stress area. It is equal to the mean of mean and root diameter.

$$d_t = \frac{d_m + d_r}{2} \quad \text{and} \quad A_t = \frac{\pi}{4} d_t^2$$

Pitch: The distance from one thread on the screw to a corresponding point on the next threaded parallel to the screw axis.

Lead: The axial distance the screw or nut travels in one revolution. It is equal to the pitch times the number of start.

L=n.p

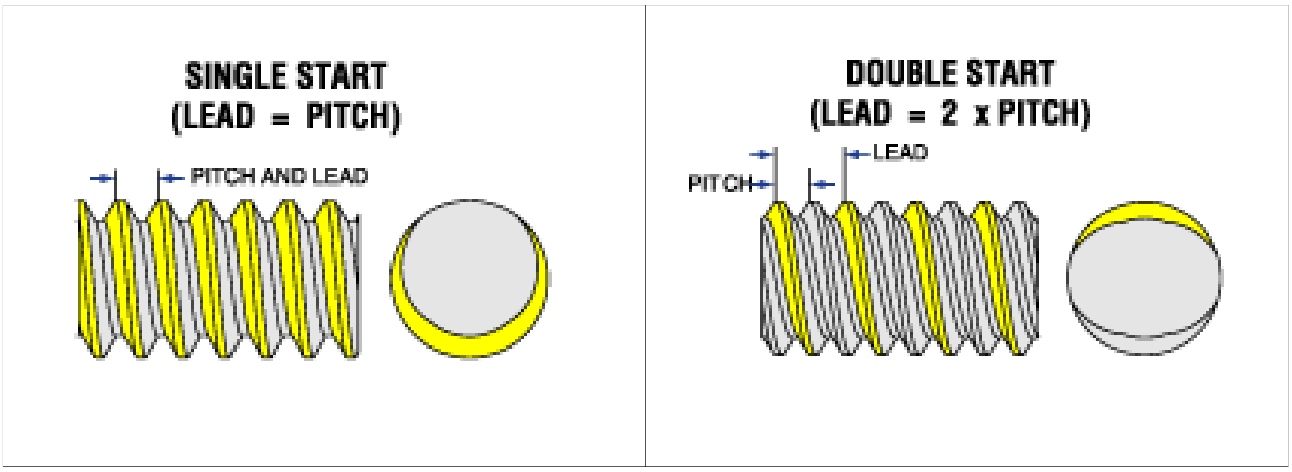
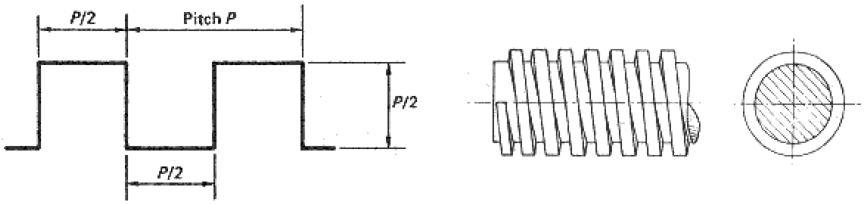


Figure 8: Number of start

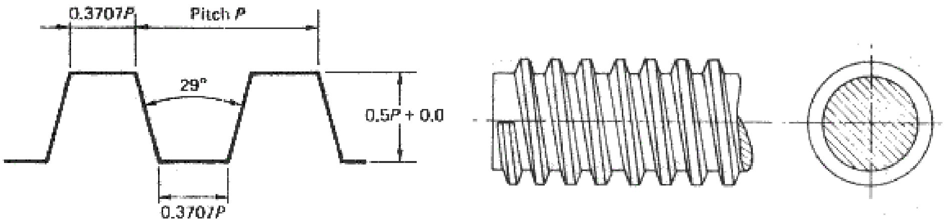
Thread Forms:

1. **Square thread form:** This form is used for power/force transmission



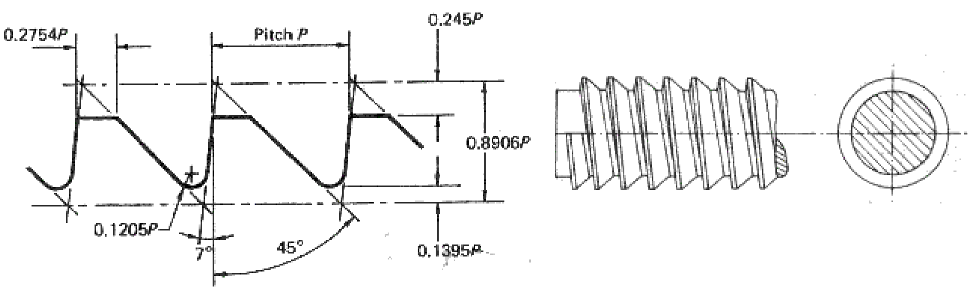
Using Areas,	Advantages;	Disadvantages;
• Linear jacks (kriko) • Clamps (mengene) • Tool carries	• Friction is low • No radial forces imposed on the mating nut • Most efficient thread form	• Difficult to machine • not suitable for using split nuts used on certain machine tool systems.

2. **Acme Thread form:** Used for power transmission



Using Areas:	Advantages:
• Lathe lead screws • Machine tool drivers	• Easier to manufacture comparing to square thread • Has superior rooth strength characteristics compared to square threads • Can be used as split nut • Thread has optimum efficiency of about %70 for helx angles between 25° and 65°. Outside this range the efficiency falls away.

3. **Buttress thread form:**
   - A strong low friction thread.
   - However it is designed only to take large loads in one direction.
   - For a given size, this is the strongest of the thread forms.
   - When taking heavy loads on the near vertical thread face this thread is almost as efficient as a square thread form

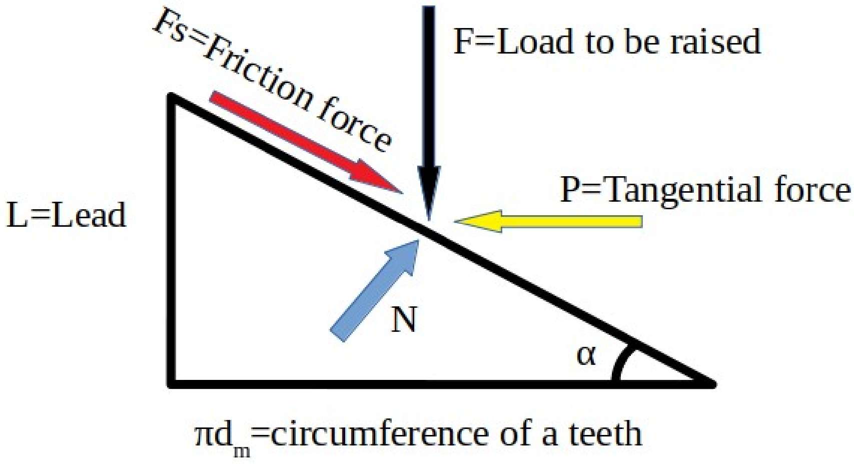


Mechanics of Power Screw:

There are three parameters as far as the mechanics of power screws are concerned,

- Torque to raising a load
- Torque to lowering a load
- Efficiency of the screw

1. **Torque for raising a load (T_R)**



ΣFx=0

P-μNcosα-Nsinα=0

=> P=μNcosα+Nsinα (1)

ΣFy=0

Ncosα-F-μNsinα=0

N(cosα-μsinα)=F

=> $N=\frac{F}{\cos(\alpha)-\mu\sin(\alpha)}$ (2)

Putting Eq.2 into Eq.1;

$$P=\mu\frac{F\cos(\alpha)}{\cos(\alpha)-\mu\sin(\alpha)}+\frac{F\sin(\alpha)}{\cos(\alpha)-\mu\sin(\alpha)} \quad (3)$$

$$\cos(\alpha)=\frac{\pi d_m}{x}$$
$$\sin(\alpha)=\frac{L}{x}$$

Putting these two into Eq.3

$$P=F\left(\frac{L+\pi d_m\mu}{\pi d_m-\mu L}\right)$$

$$T=P\frac{d_m}{2}$$

we get; $$T_R = \frac{F d_m}{2} \left(\frac{L + \pi d_m \mu}{\pi d_m - \mu L} \right)$$

or substituting $\tan(\alpha) = \frac{L}{\pi d_m}$

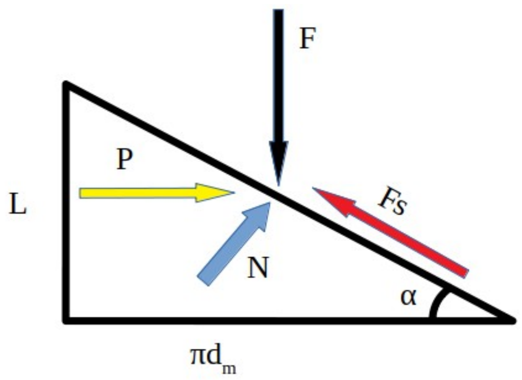
we get; $$T_R = \frac{F d_m}{2} \left(\frac{\tan(\alpha) + \mu}{1 - \mu \tan(\alpha)} \right)$$

or by considering $\mu = \tan(\rho)$

we get; $$T_R = \frac{F d_m}{2} \tan(\alpha + \rho)$$

any of these 3 formulations can be used for calculating T_R (Torque for raising a load)

2. Torque for lowering a load (T_L)



From the same considerations (force equilibriums);

$$T_L = \frac{F d_m}{2} \left(\frac{\pi d_m \mu - L}{\pi d_m + \mu L} \right)$$

$$T_L = \frac{F d_m}{2} \left(\frac{\mu - \tan(\alpha)}{1 + \mu \tan(\alpha)} \right)$$

$$T_L = \frac{F d_m}{2} \tan(\rho - \alpha)$$

any of these 3 formulations can be used for calculating T_L (Torque for lowering a load)

3. Efficiency (ε)

Efficiencies between 20% and 70% are obtained with conventional poer screws. The efficiency is considered for the condition of load raising.

$$\varepsilon=\frac{T_o}{T_R}$$

=> To=torque when there is no friction.

$$T_o=\frac{FL}{2\pi}$$

If the collar friction is negligable;

$$\varepsilon=\frac{\tan(\alpha)}{\frac{\mu+\tan(\alpha)}{1-\mu\tan(\alpha)}} \quad \text{or} \quad \varepsilon=\frac{\tan(\alpha)}{\tan(\alpha+\rho)}$$

ρ = friction angle

Self Locking:

In the design of power screws, it is demanded that the screw does not lower itself without any external effect. This condition is self-locking condition.

In order to provide this condition, the coefficient of friction must be greater than the tangent of the lead angle.

If $\mu \geq \tan(\alpha)$ or $\mu \geq L/\pi d_m$ or $\rho \geq \alpha$

=> Then the screw is self-locking.

Figure 10 Self-Locking condition

Collar Condition:

When the screw is loaded axially, a thrust or collar bearing must be employed between the rotating and stationary members in order to carry the axial component. When it is taken into consideration (the torque due to collar is added);

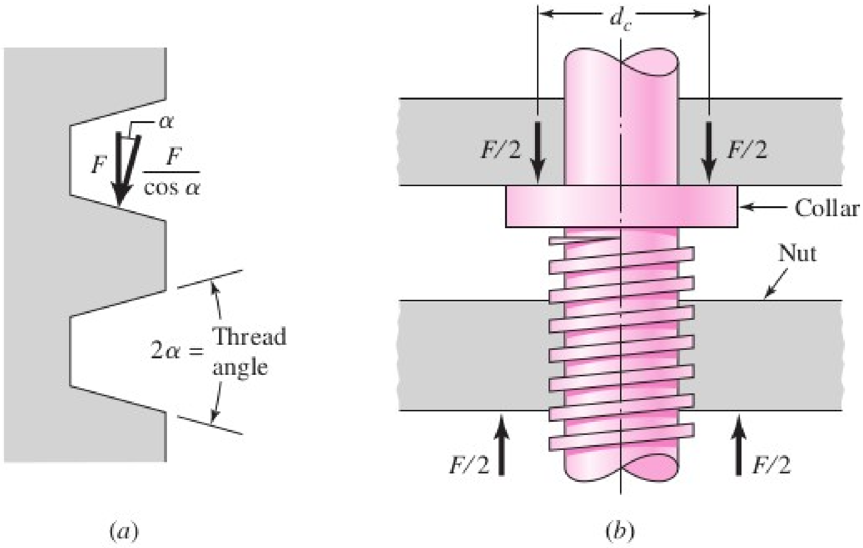
$$T_R = \frac{F\,d_m}{2}\tan(\alpha+\rho) + \frac{\mu_c d_c F}{2}$$

$$T_L = \frac{F\,d_m}{2}\tan(\alpha-\rho) + \frac{\mu_c d_c F}{2}$$

d_c=collar diameter
μ_c=collar friction

Figure 8–7

(*a*) Normal thread force is increased because of angle α; (*b*) thrust collar has frictional diameter d_c.



Highlights:

- In order to reduce the cost of wear, nuts with softer materials are recommended.
- Same material for the screw and the nut should be avoided especially under severe operating condition because of galling (kırıcı) action between the screw/nut interface
- High friction means low efficiency. In order to increase efficiency, balls are introducing between the threads of the screw and the nut (Recirculating ball screw).

RBS isfrequently used in;

- CNC tools,
- High accuracy positioning servo systems

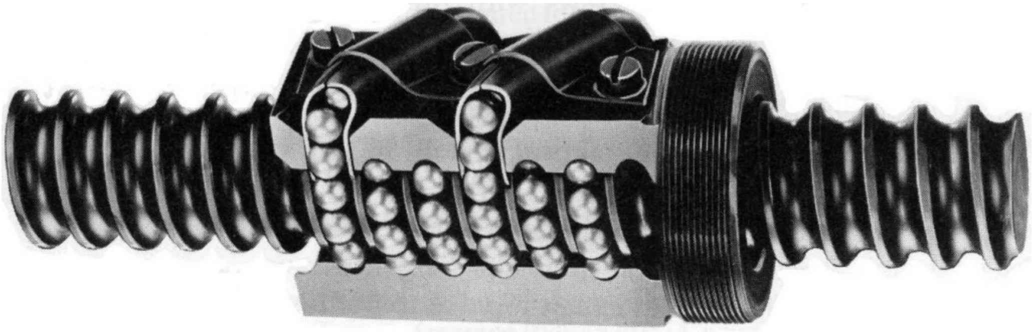


Figure 11: Recirculating screw balls

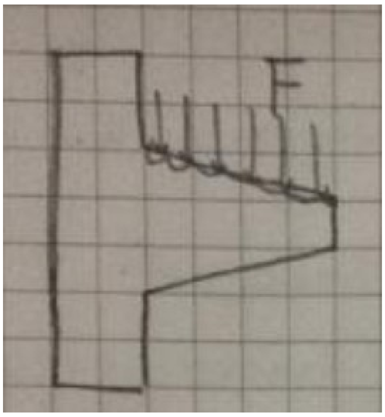
Advantages of RBS:	Disadvantages of RBS:
• Smooth movement • No stick-slip • Easily preloaded for eliminating backlash	• Rigidity • Needing higher level of lubrication • Additional brakes for locking

Strength of Power Screws:

1. For the screw and nut threads
   - Bearing stress
   - Shear stress
   - Bending stress
2. For the screw body
   - Tensile or compressive stress
   - Torsional stress
   - Combined loading
   - Buckling

1. **For the screw and nut threads:**

   - Bearing stress (ezilme):

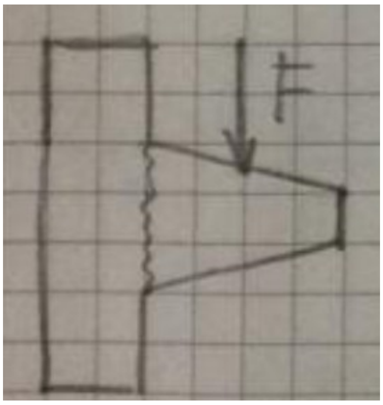


$$\sigma_b = \frac{2F}{\pi d_m h} \qquad n = \frac{\sigma_{b(allow)}}{\sigma_b}$$

σ_b is dependent on the speed of the load and screw&nut materials

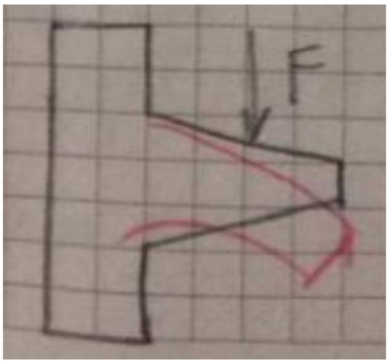
h, nut height

   - Shear Stress:



For Screw:	For Nut:
$\tau_s = \frac{2F}{\pi d_r h}$ $n = \frac{S_{sy}}{\tau_s}$	$\tau_n = \frac{2F}{\pi d h}$ $n = \frac{S_{sy}}{\tau_n}$

   - Bending stress



$$\sigma = \frac{6F}{\pi d_m h} \qquad n = \frac{S_y}{\sigma}$$

h, nut height

2. **For the screw body:**

- **Tensile or compressive stress**

$$\sigma_x = \frac{F}{A_t} \qquad A_t = \frac{\pi d_t^2}{4} \qquad d_t = \frac{d_r + d_m}{2}$$

d_t=tensile strength area

Note: If a threaded rod is subjected to pure tensile loading; one might expect its strength to be limited by the area of its minor diameter. However testing shows that, their tensile strength is better defined by the average of the minor and pitch diameter.

- **Torsional stress**

$$\tau_{xy} = \frac{16\,T_R}{\pi d_t^3} \qquad n = \frac{S_{sy}}{\tau_{xy}}$$

d_t=tensile strength diameter.

- **Combined loading;**

If axial and torsional stress is acting together,

Based on DET;	Based on MSST;
$\sigma' = \sqrt{\sigma_x^2 + 3\,\tau_{xy}^2}$ $n = \frac{S_y}{\sigma'}$	$\tau_{max} = \frac{1}{2}\sqrt{\sigma_x^2 + 4\,\tau_{xy}^2}$ $n = \frac{S_{sy}}{\tau_{max}}$

- **Buckling of screw**

If Lscrew > 8 d_r ; buckling consideration is taken into account

if L/k < 100 ; Johnson’s column (short column)

if L/k > 100 ; Euler’s column (long column)

“I (moment of inertia) and k (radius of gyration) is calculated based on d_r”

ME 307 – Machine Elements I

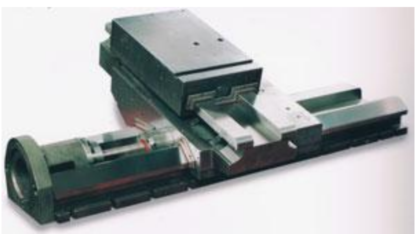
Chapter 8

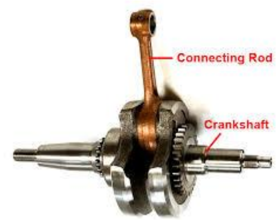
Design of Bolted Joints

Mechanical Engineering
University of Gaziantep

Dr. Sadık Olguner
Dr. Hakan Çandar

- In machines, the parts are joined together either to form **sliding joints** or **fixed joints**.
- The **sliding joints** include joints which link for example *a slide and slideway, a shaft and a bearing, a shaft and a gear or pulley, a crank and a connecting rod, a crank pin and a connecting rod*. In this group of joints, **parts move relative to each other**.

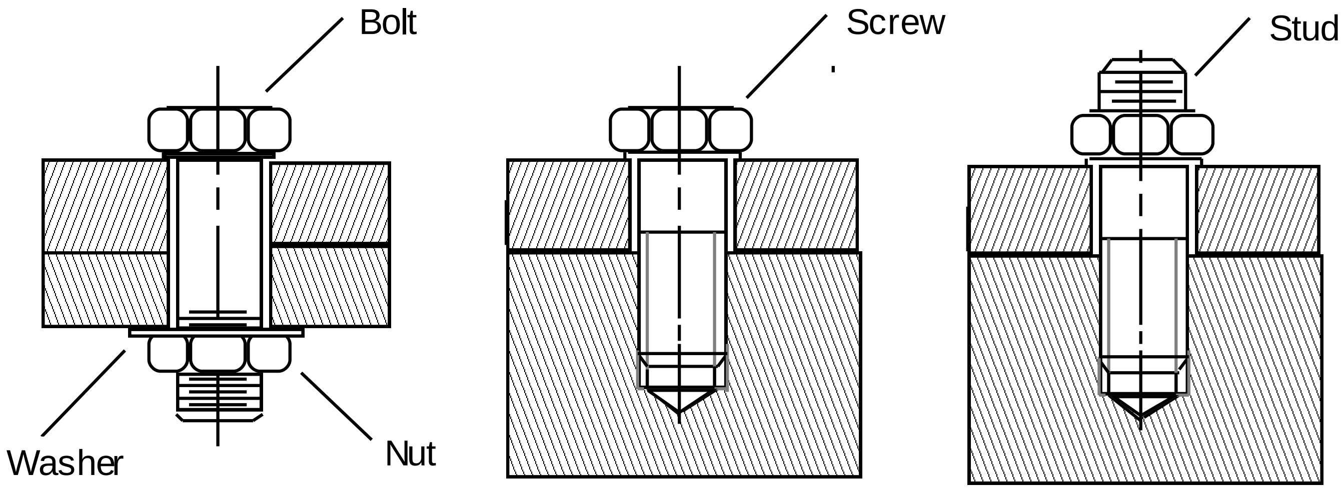




- The **fixed joints** include the joints which are obtained by **fastening the parts** such as *a cylinder and its cover, the plates of a boiler, etc..* In this group of joints, **there is no relative movement between the parts**.

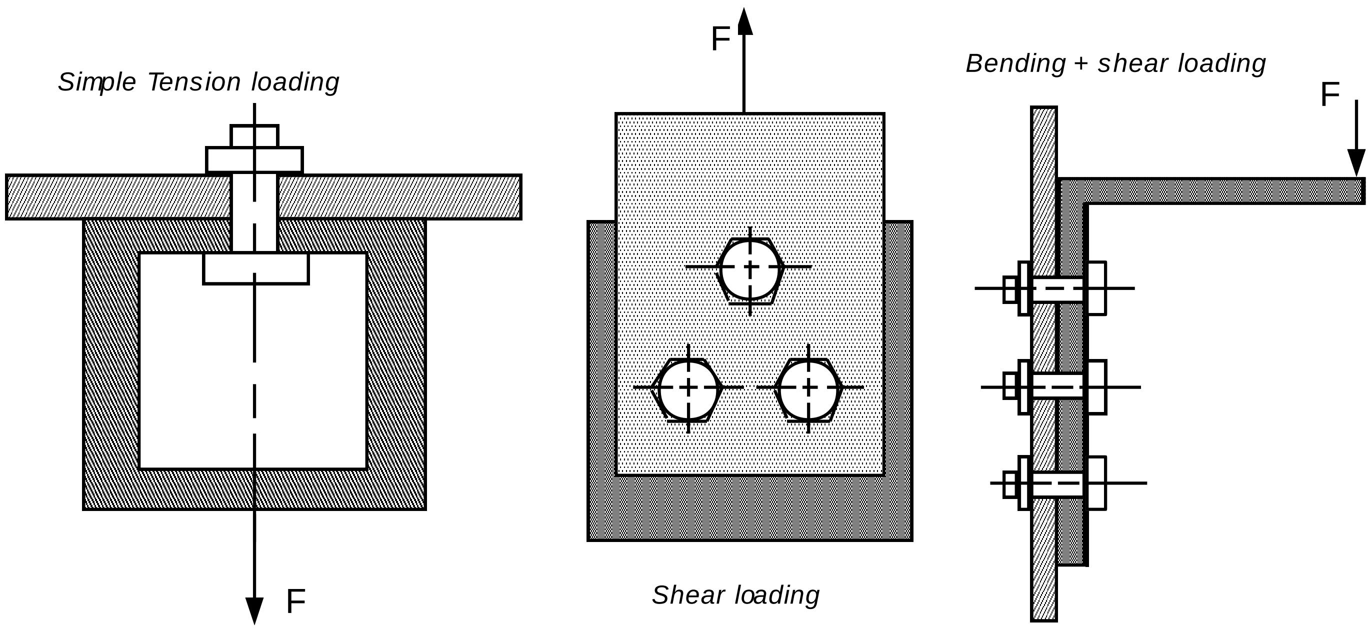


- The main standard threaded fasteners include **bolts**, **studs**, and **screws**. A bolt is a threaded element which is used with always a nut. They are produced in standard sizes. These elements are used in combination with washers and retainers of various design which help to prevent loosining of threaded elements.



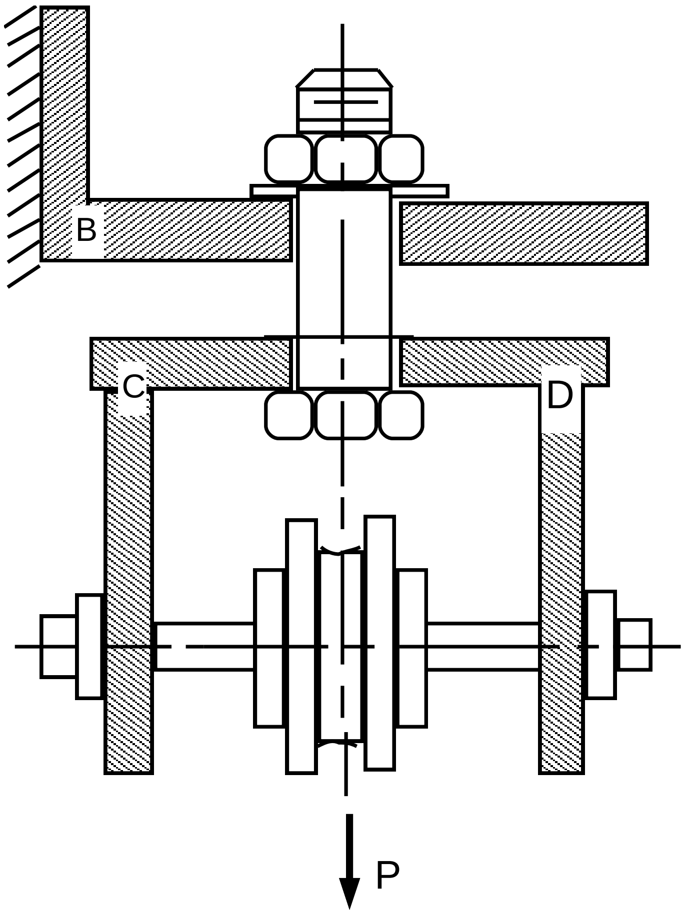
- A bolt is tightened by exerting torque on the nut. If a connection is desired which can be disassembled without destructive methods then, a bolted joint will be a good solution. These joints may be subjected to **tensile loads** and **shear loads** or **combination of these**.

- Typical joint configurations where the threaded fasteners are loaded in **simple tension**, **shear** or **combined shear and bending** are shown in the following figures:

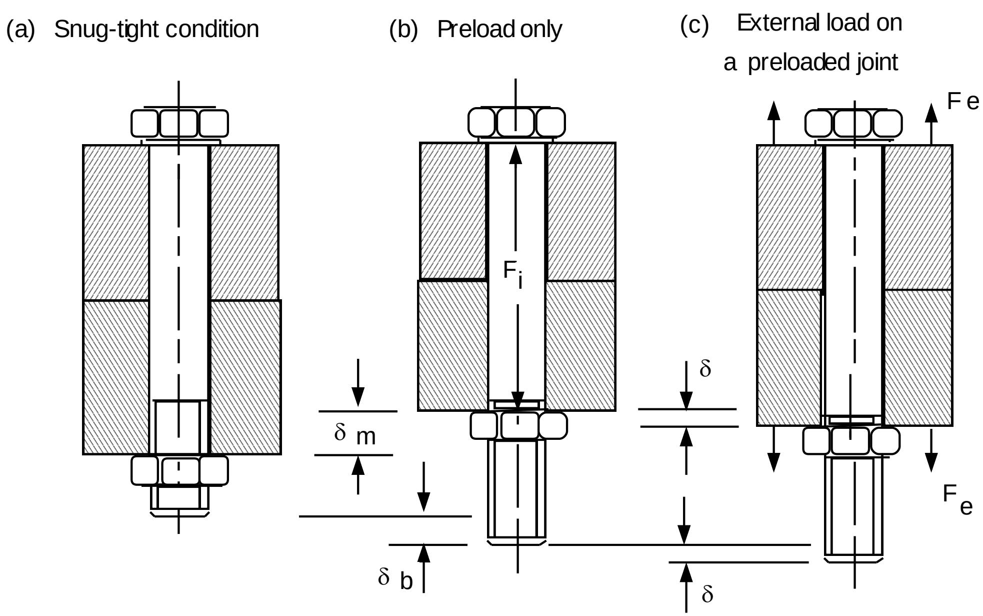


- This is the case where the **dominant load acts paralel to the axes of bolts**. In the joint shown below, there is no preload and the axial load is totally carried by the bolt. The bolt may fail **by yielding** or **by shearing off** the threads of the bolt and the nut.

- **Normal tensile stress** in the body of the bolt is to be compared with the yield strength of the material of the bolt. **Shear stresses on the threads** of the bolt and the nut are calculated in a similar manner described in the previous section. The diameter of the bolt, and the nut height are determined by using these stresses.



- In order to place the bolted members in compression for better resistance to the external tensile loads and to create friction force between parts to resist the shear load and hence to provide uniform shear load distribution between the bolts, **bolts are usually preloaded by tightening the nuts**. The following figures represent the three conditions of a bolted joint:

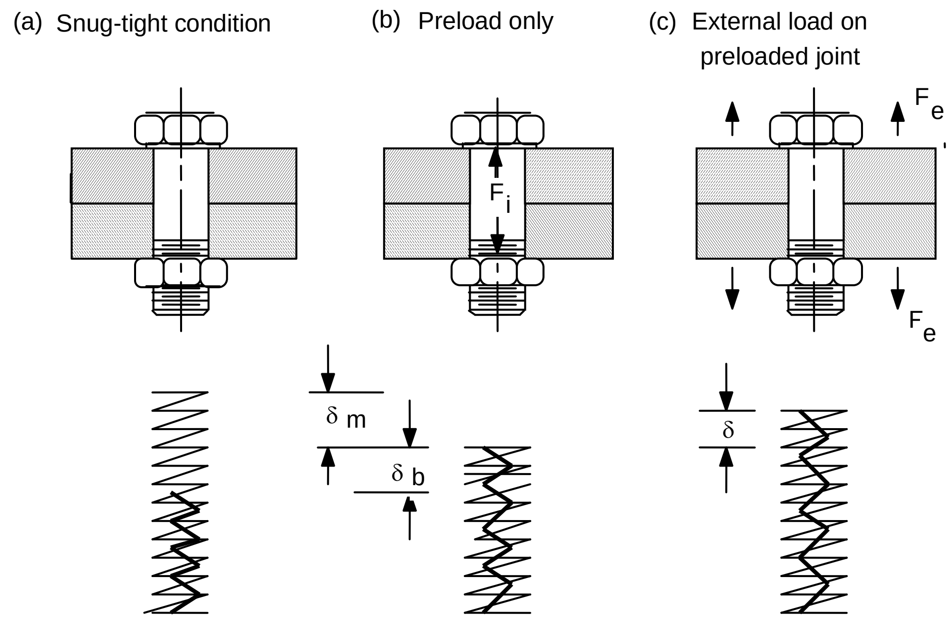


(a) when the nut has been hand tightened so that the components are snug tight before external load is applied. (In this case, the nut is turned until the clearance between the members is totally removed but there is no deflection of the members and the bolt)

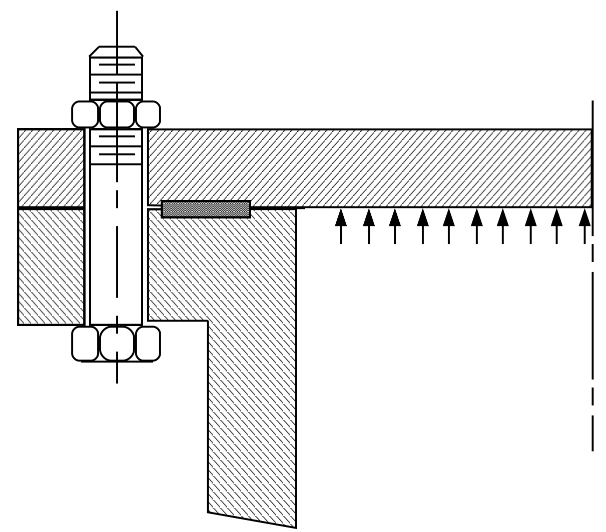
(b) when the nut has been tightened to produce a preload (pre-tension, initial tension) F_i in the bolt *(any extra turn after snug tight condition is going to produce tension in the bolt and compression on the members)*, the bolt stretches and the joint members are compressed under the influence of forces applied at the nut and bolt faces. The compressive load applied to the members is clearly equal to the bolt tension F_i, so that the extension of the bolt and the compression in the members will be inversely proportional to their relative stiffnesses.

(c) when an external load F_e is applied to the joint, the bolt will extend extra with an amount of δ while compression on the members is relieved with an amount of δ. This condition is the result of compatibility condition *(elements are in contact)*.

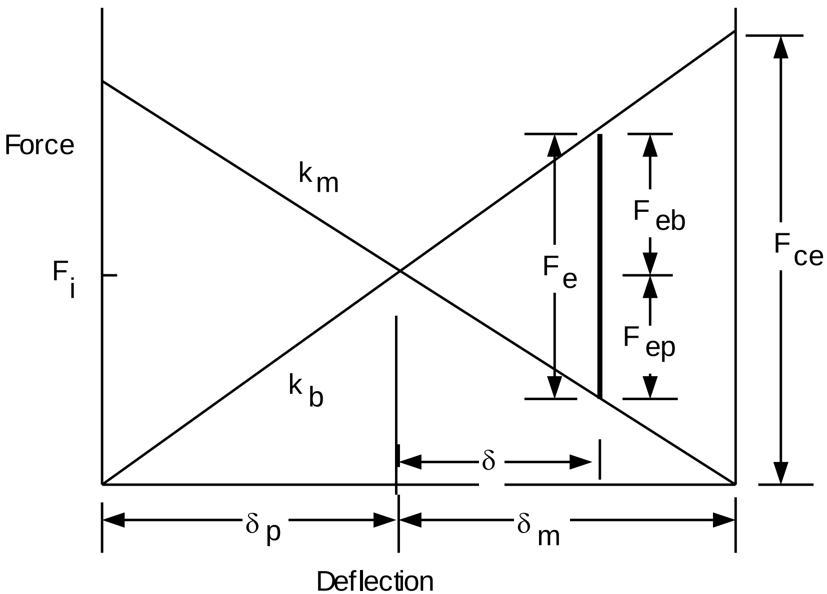
- The mechanism of preloading, load shared by the bolt and compressed members and the effect of external load can also be explained by using spring analogy.
- Bolts and joint members deform elastically when the bolt is tightened. In effect, **they act like springs** *(deflection characteristics of bolt and members are represented by an extension spring and compression springs respectively)*.



- A typical application of preloaded joint is a hydraulic cylinder head. Bolts are preloaded and the external load on the bolts are created by the pressure inside the cylinder.



- The stiffnesses of the bolt and the members play the vital role on the determination of the resulting loads on the bolt and on the members. The way of load sharing is illustrated in Figure below.



$$F_e = F_{eb} + F_{ep}$$

Stiffness ratio $C = \frac{k_b}{k_b + k_m}$

$$F_{eb} = CF_e \quad \text{and} \quad F_{ep} = (1-C)F_e$$

The loads on the bolt and on the members are

$$F_b = F_i + CF_e \qquad F_m = F_i - (1-C)F_e$$

Stiffness of the bolt

$$k_b = \frac{A_b E_b}{L}$$

Stiffness of the members are calculated by assuming that the parts in the joint are assumed to act like springs connected in series

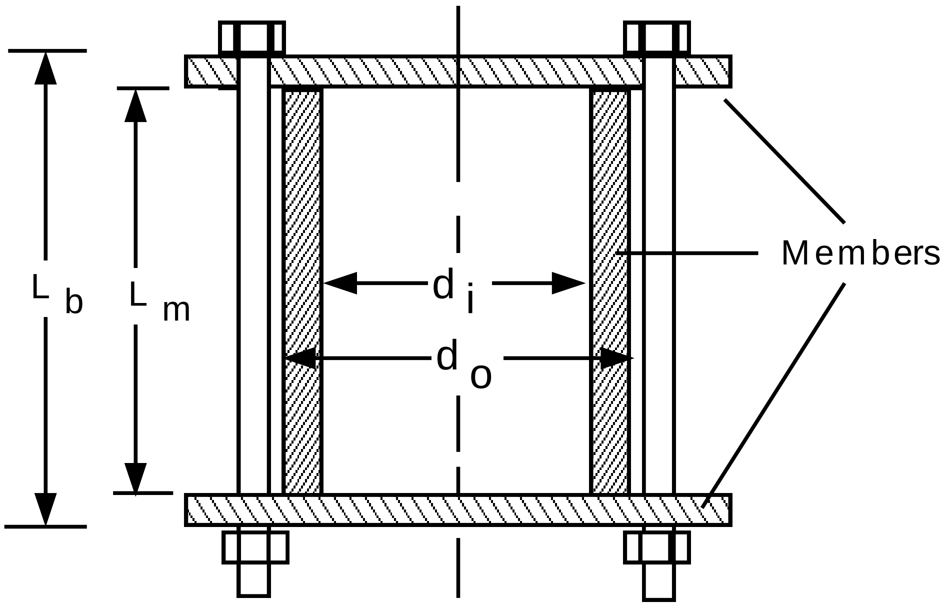
$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots\ldots + \frac{1}{k_n}$$

Determination of each one of these stiffness components is difficult due to non-uniform pressure distribution in these members. *Size of the members, thickness of the members, diameter of bolt are the effective parameters in compression distribution.*

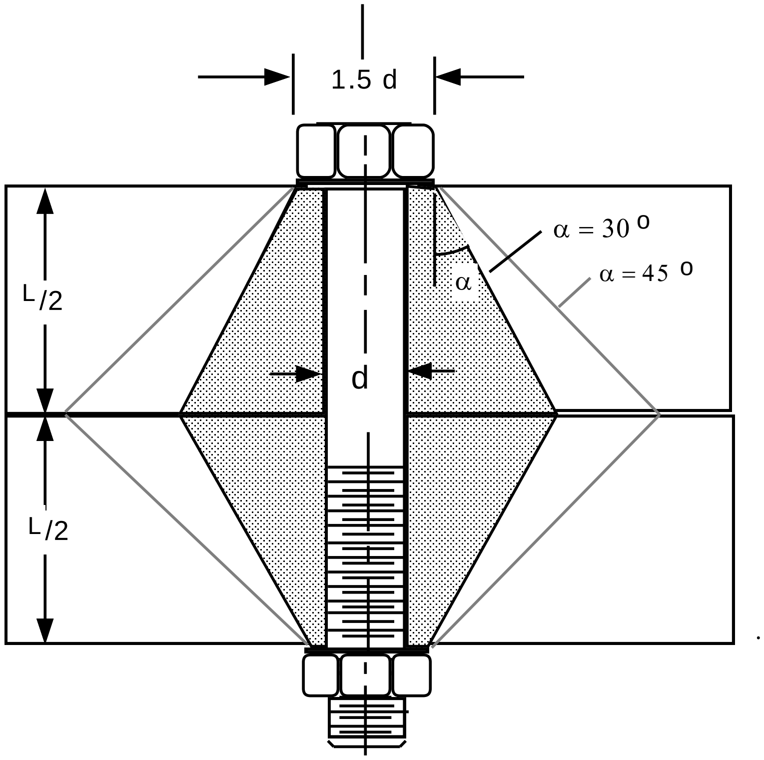
- In the following application, we have three members; namely an open ended cylinder and two heads. Variation of compression in radial direction can be considered negligible and therefore, compressed area $\boldsymbol{A_m}$ is taken as the cross sectional area of the cylinder. If the thicknesses of the cylinder heads are comparably small with respect to cylinder length, then, the member stiffness $\boldsymbol{k_m}$ is calculated as:

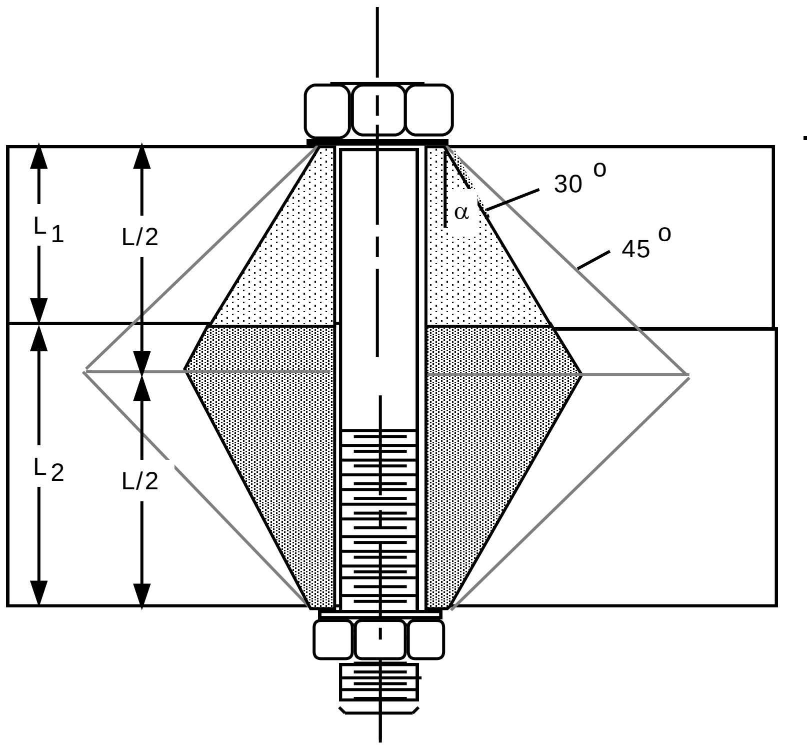
$$k_m = \frac{A_m E_m}{L_m}$$

where $\boldsymbol{E_m}$ is modulus of elasticity of the material of the cylinder and $\boldsymbol{A_m} = \pi(d_o^2 - d_i^2)/4$.



- In the literature, there were many attempts for the determination of member stiffness. The main difference between them is in the assumption of cone angle ***α***. Most of the bolted joints are similar to the following application (*left-hand side*) where we have conical compression distribution on the members. For different thicknesses, spreading out of the compression is shown in the *right-hand side*.





- **Shigley and Mishke** suggest the following formula ***for different cone angle (α)***.

For the assumption of 30º

$$k_i = \frac{1.813 E_i d}{In\left(5\frac{1.15L_i + 0.5d}{1.15L_i + 2.5d}\right)}$$

Stiffness of two members, $\boldsymbol{k_m}$ is determined as:

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2}$$

If the parts are made from same materials
(E=E₁=E₂)
and have equal thickness
(L_1=L_2=L/2)
then the overall stiffness $\boldsymbol{k_m}$ will be

$$k_m = \frac{1.813 Ed}{2In\left(\frac{2.885L + 2.5d}{0.577L + 2.5d}\right)}$$

For the assumption of 45º

$$k_i = \frac{\pi E_i d}{In\left(\frac{5(2L_i + 0.5d)}{2L_i + 2.5d}\right)}$$

Stiffness of two members, $\boldsymbol{k_m}$ is determined as:

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2}$$

If the parts are made from same materials
($E=E_1=E_2$)
and have equal thickness
($L_1=L_2=L/2$)
then the overall stiffness $\boldsymbol{k_m}$ will be

$$k_m = \frac{\pi Ed}{2In\left(5\frac{L + 0.5d}{L + 2.5d}\right)}$$

- **Wileman et al**. and **Filiz et al**. had made considerable contribution by using finite element method and suggested the following formulas.

Wileman et al.

$$k_m = EdA_i e^{\left(B_i d/L\right)}$$

where ***E*** is modulus of elasticity of the member and ***d*** is bolt diameter

where ***A*** and ***B*** are constants related to the material.

For steel A_i *=0.78715 and* B_i *=0.62873*
For aluminum A_i *=0.79670 and* B_i *=0.63816*
For gray cast iron A_i *=0.77871 and* B_i *=0.61616*

Filiz et al.

$$k_m = \frac{\pi}{2} E_{eq} d \left(\frac{1}{1-B_2}\right) e^{\left(\frac{\pi}{5}-B_1\right)\left(\frac{d}{L}\right)}$$

where ***E*** is equivalent modulus of elasticity of the members and ***d*** is bolt diameter

$$E_{eq} = \frac{E_1 E_2}{E_1 + E_2}$$

$$B_1 = \left(\frac{0.1d}{L}\right)^2 \qquad B_2 = \left(1-\frac{L_1}{L_2}\right)^8$$

This formula can be used for quick calculations where we have different materials and different thicknesses.

- In preloaded joints, the problem is how to determine the value of preload to ensure that parts (joint faces) will not be seperated on the one hand and the bolt will not fail by tension on the other hand. These two conditions can be named as **strength** and **no seperation (sealing) requirements**.
- **Strength requirement dictates that** load on the bolt must be such that the resulting stress must be smaller or equal to the yield strength *(or proof strength,* S_p*, which is defined as the limiting value of the stress at which there will be no permanent deformation)* of the bolt material.
- **Sealing requirement dictates that** there must exist a compressive load on the members *(compressive load on members must not be totally relieved)*. For static loading, they are expressed as:

$F_b \leq S_y A_t$ or $F_b \leq S_p A_t$ where $S_p = 0.85 S_y$

$$F_m \geq 0$$

- The bolt and member force inequalities can be combined to provide an acceptable range for the preload as:

$$(1-C)nF_e \le F_i \le A_t S_p - CnF_e$$

where ***n*** is the load factor of safety. If **the strength factor of safety is included**, then above inequality may be written as:

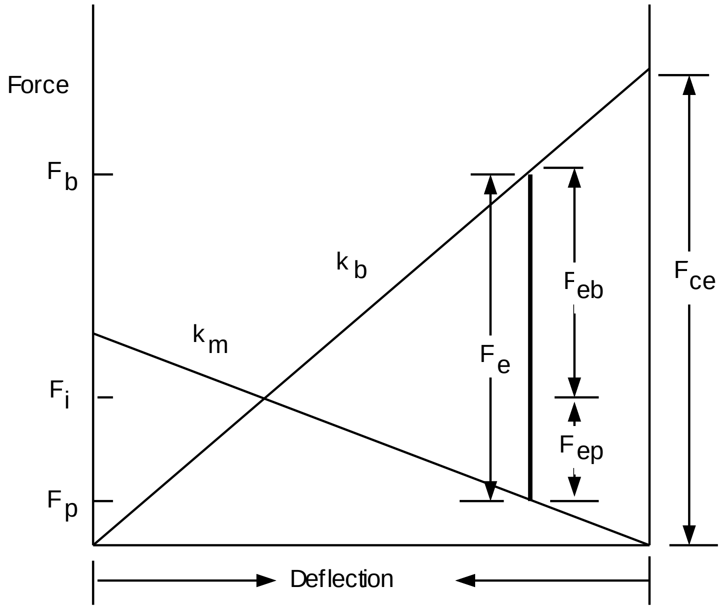
$$(1-C)nF_e \le F_i \le \frac{A_t S_p}{n_s} - CnF_e$$

The critical load, $\boldsymbol{F_{ce}}$ is calculated by equating $\boldsymbol{F_m}$ equal to zero.

$$F_{ce} = \frac{F_i}{1-C}$$

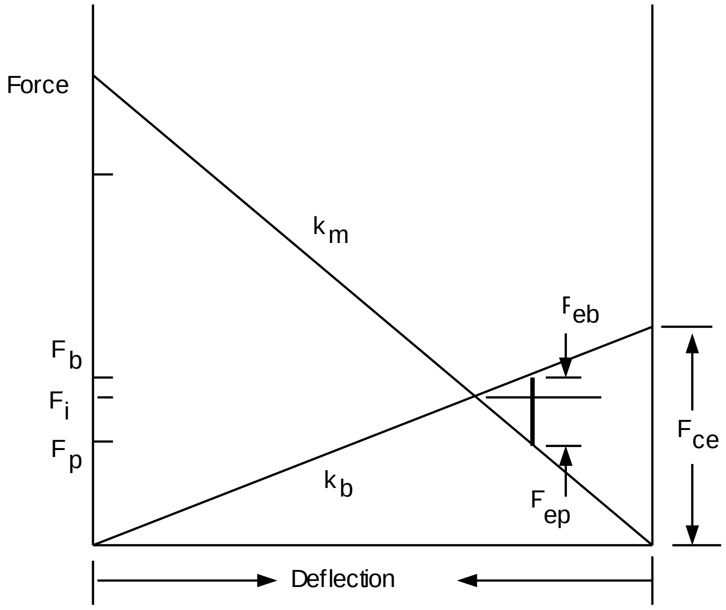
If the bolted joint is to be designed such that **the sealing is of prime importance**, ***C*** must be large that the stiffness of the members must be comparably smaller than the stiffness of the bolt.

Force Deflection Diagram for $k_b >> k_m$



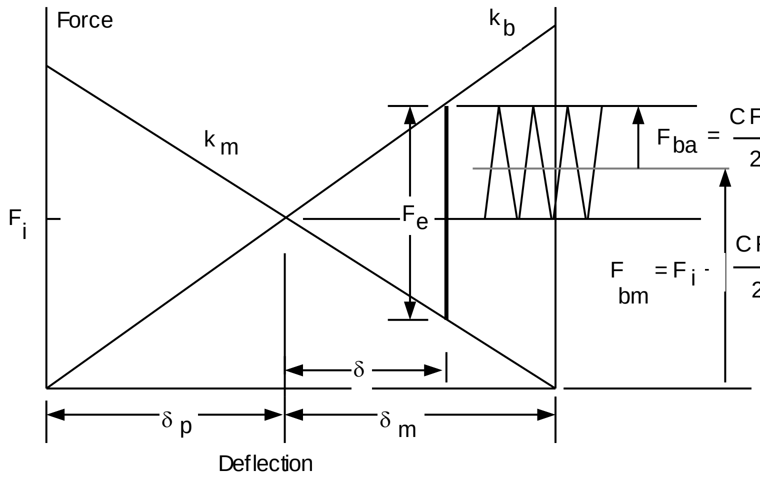
If strength of the joint is the governing design criterion, ***C*** value must be small. That is, the stiffness of the bolt is required to be comparably smaller than the stiffness of the members.

Force Deflection Diagram for $k_m >> k_b$



*Whichever is of prime importance, **both requirements must be satisfied** and it is the designer's job to find a compromise between these two conflicting conditions.*

- If the joint is subjected to a dynamic load as shown in the following figure, **fatigue failure may be of concern**.



Alternating and mean stress components are:

$$\sigma_a = \frac{CnF_e}{2A_t} \qquad \sigma_m = \frac{F_i}{A_t} + \sigma_a$$

By using one of the fatigue failure theories, strength factor of safety may be calculated.

Modified Goodman criterion:

$$n_s = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u}}$$

The maximum value of preload that could be assigned is determined as:

$$F_i = \frac{A_t S_u}{n_s} - \frac{CnF_e}{2}\left(\frac{S_u}{S_e} + 1\right)$$

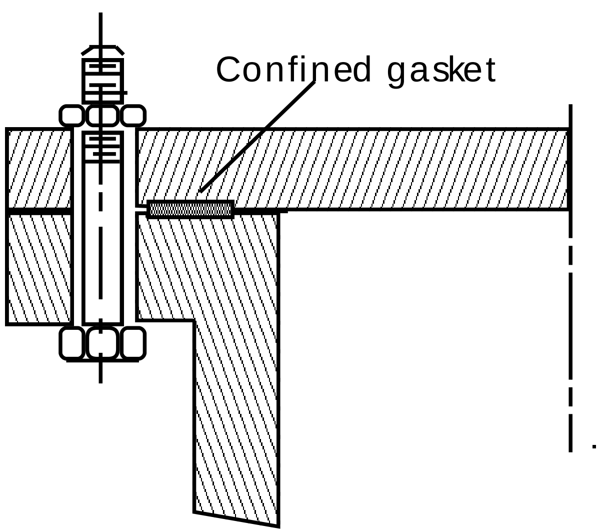
Bolt is subjected to axial loading, therefore the problem is axial fatigue. Endurance limit is calculated as:

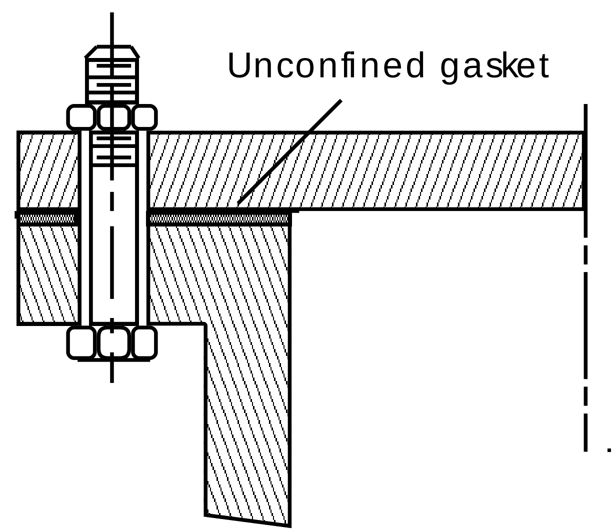
$$S_e = k_a k_b k_c k_d k_e S_e'$$

where $\boldsymbol{k_a=1}$, $\boldsymbol{k_b=1}$ and $\boldsymbol{S_e'=0.45S_{ut}}$. Modifying factor for stress concentration is $\boldsymbol{k_e=1/K_f}$ where $\boldsymbol{K_f}$ is dependent on the type of production of threads of bolt. It includes the effect of notch sensitivity and surface finish. Therefore, endurance limit formula reduces to

$$S_e = k_c k_d \frac{1}{K_f} S_e'$$

- Gaskets inserted between the connected parts are of great help **for sealing requirement**. In the case of **confined gasket**, the gasket will not contribute any load sharing, while **in unconfined gasket** applications, the gasket will behave as the third part in the connection and its contribution to load sharing is dependent on the modulus of elasticity of gasket material.
- If the modulus of elasticity of gasket material *(like cork, compressed asbestos etc.)* is very small compared to modulus of elasticity of the connected parts, then stiffness of gasket will be taken equal to k_m and it is considerably small with respect to k_b which is good for sealing requirement.





- **In gasketed joints**, in addition strength requirement under static and fatigue loading, there are **two additional requirements**.

1) The preload must be large enough to achieve minimum sealing pressure required for the given gasket material.

$$F_i \geq A_g \, p_o$$

where $\boldsymbol{A_g}$ is the gasket area and $\boldsymbol{p_o}$ is minimum gasket seal pressure

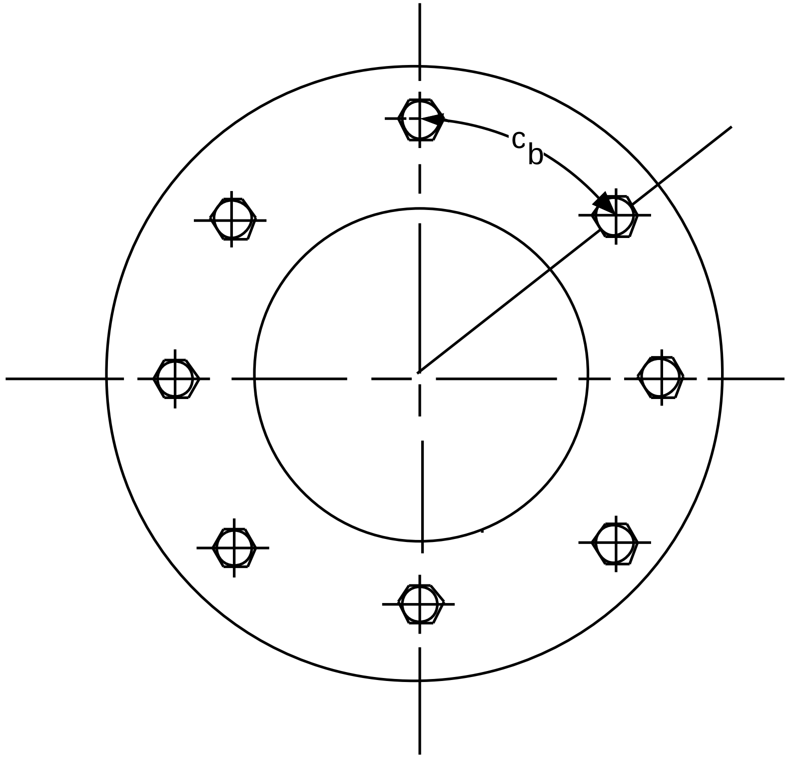
2) The compressive load on the members must not be relieved totally. In other words, compressive load on the members must satisfy the following inequality.

$$F_m \geq A_g m p$$

where ***m*** is gasket factor *(like factor of safety varies from 2-4 for different gasket materials)*, ***p*** is the actual (working) pressure in pressure vessels (cylinders).

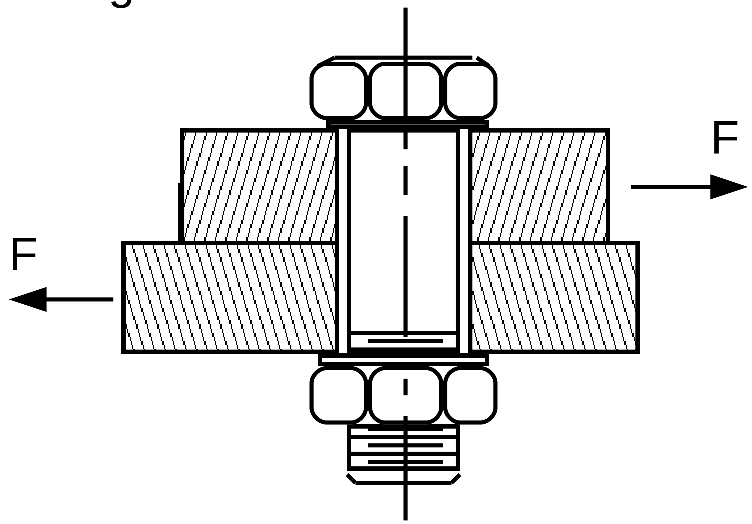
- **In pressure vessel applications**, the spacing of the bolts is also important and the bolts should not be spaced more than 10 bolt diameters apart to obtain uniform load distribution on the members and the gasket.
- And for tightening with open-wrench, the space between the bolts should not be less than 3.5 bolt diameters.

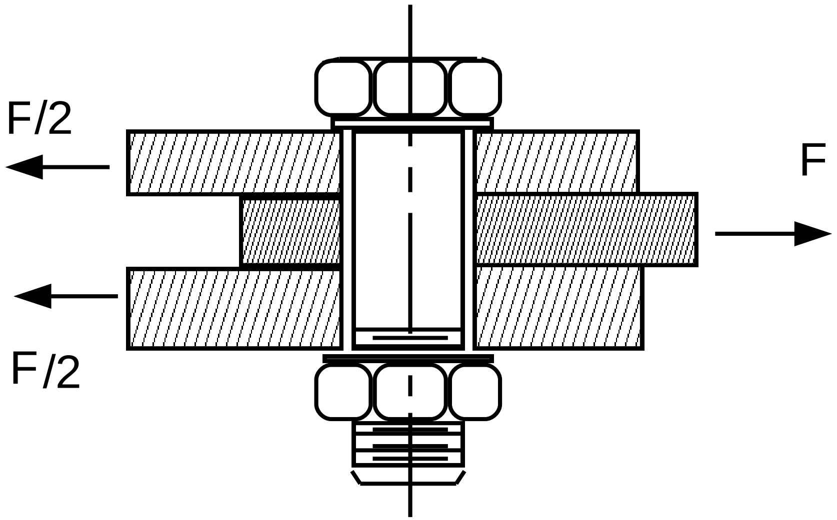
$$3.5d \le c_b \le 10d$$



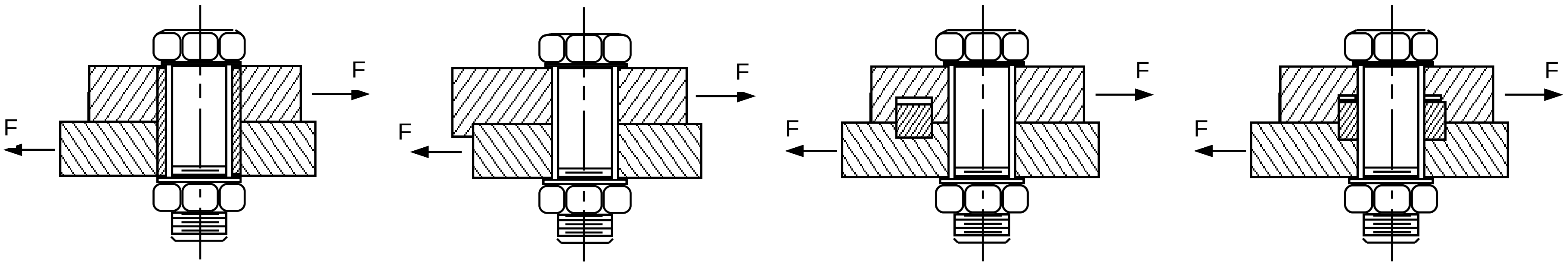
From the above discussions, **design study of preloaded bolted joints subjected to tensile loading requires** determination of the preload, the bolt size, the number of bolts, the type of gasket material *(if used)* and the grip length in such a way that both strength and sealing requirements are both met reasonably.

- Strength of bolts withstanding direct shear loading is to be treated in the similar manner of riveted joints loaded in shear and explained in the next chapter. In bolted joints, there is a clearance between the hole and the bolt. This may result in non-uniform distribution of load. When the bolts fit the hole with a clearance as shown in figures below, the joint may be tightened prior to the application of shear force in order to prevent relative displacement of the elements in the direction of acting forces.

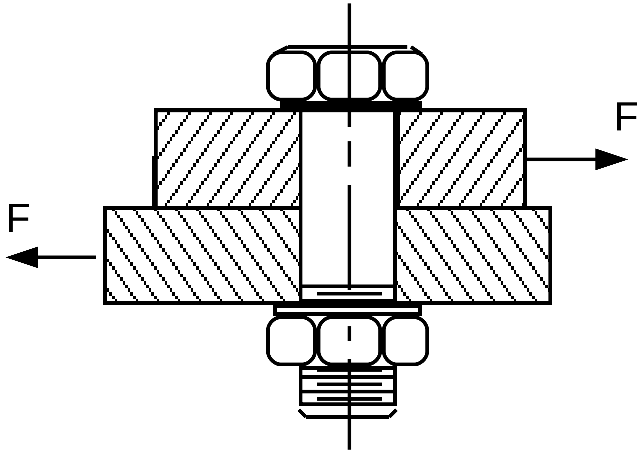




- These shear forces may as well create bending moment. Sliding of the clamped components laterally, due to shear force F, should be prevented by separate (special) means to prevent relative movement of parts and relieve bolt from shear forces as shown in figures below.



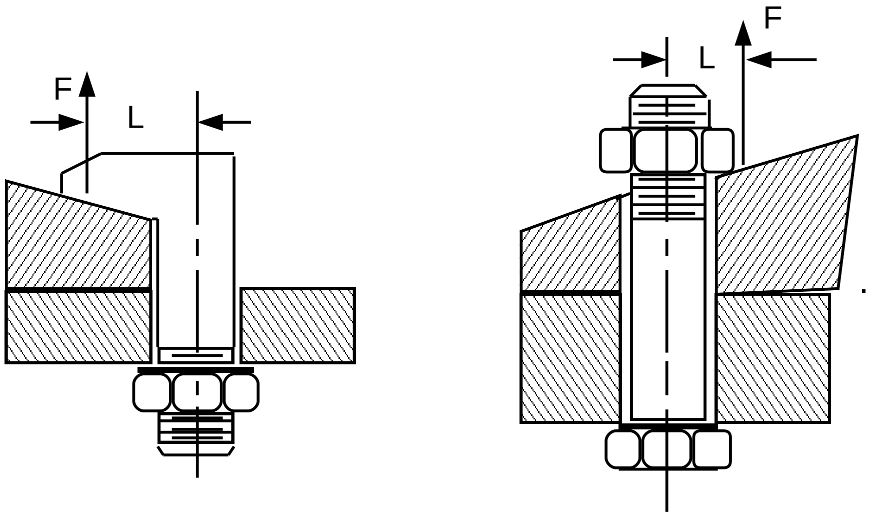
- When the bolt is used in a hole with a small interference or with a zero clearance, the shear load is taken by the bolt body. Such a joint is not necessarily to be preloaded. In this case, **shear stress in the bolt body is to be compared with the yield strength in shear**.



- In some cases the joints may be subjected to bending forces as shown in the following figure. Bending forces may arise from specific features in the structure, from misalignments of contact surfaces of the bolt head, the nut and the connected parts.
- Bending moment may be created due to asymmetric head or the surfaces of the parts and the bolt may not have a full contact.

The stress arising in bolt section is:

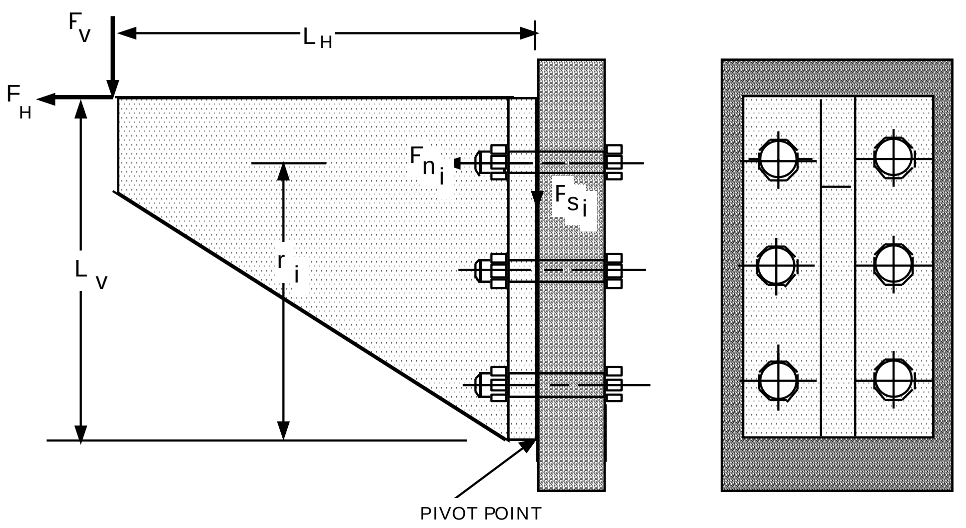
$$\sigma_{net} = \sigma_F + \sigma_b = \frac{F}{A_t} + \frac{Mc}{I}$$



(a) (b)

Bending & Shear Loading

- Consider a bracket connected to a wall with certain number of identical bolts as shown in the figure below. When force components in horizontal and vertical directions are applied, determination of size of identical bolts requires the determination of forces acting on each bolt. The worstly loaded bolt will be considered as critical bolt for which the size is to be determined.
- Analysis of the forces on each bolt is given below based on the assumption that bracket is completely rigid and under the effects of forces, bracket is assumed to rotate about the pivot point.



Each Bolt withstands a shear Force

$F_{si} = F_v / (N)$

The resulting bolt shear stress

$\tau = F_{si} / A$

Each bolt withstands a tensile force of

$F_{ni} = (M.\, r_i)/(r_1^2 + r_2^2 + \ldots + r_N^2)$

Bending moment about pivot point

$M = F_v.\, L_h + F_h.\, L_v$

The resulting tensile bolt stress

$\sigma = F_{ni}/A$

- Assuming all stresses developed only as a result of bracket loading i.e zero preload and zero residual bolt torque, **Von-Mises stress** in the bolt resulting from combined loading is calculated and factor of safety is determined.

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

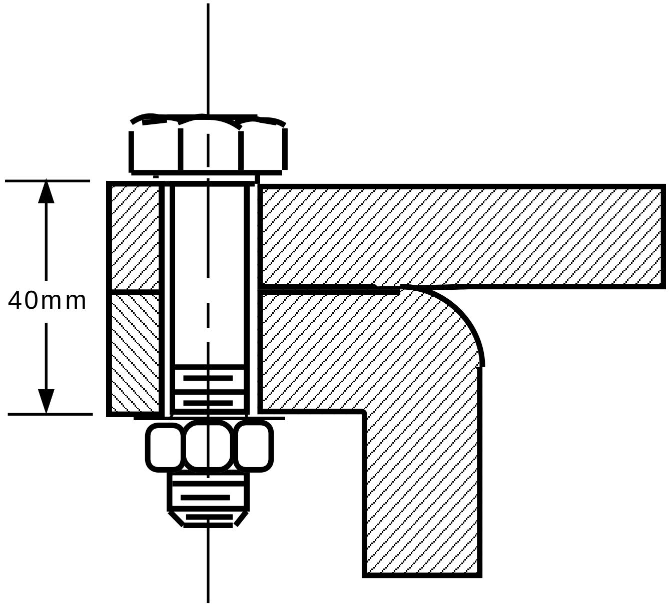
- In order to estimate the design factors of safety it is necessary to consider the failure modes. The preferred failure criteria for ductile metals is the Distortion Energy Theory. For a stress state associated with a bolt i.e pure tensile stress $\boldsymbol{\sigma_x}$ combined with shear stress $\boldsymbol{\tau_{xy}}$, the factor of safety relative to the material tensile strength $\boldsymbol{S_y}$, is calculated as $\boldsymbol{n = S_y / \sigma'}$.

- A section of a large coupling is shown below. Each bolt in this joint experiences a repeated load of 20 kN. The members are made from steel and have equal thicknesses, and each bolt is initially tightened carefully to 60% of proof load.

  The bolt is to be Metric grade 8.8 M14x1.5 (fine pitch series). The threads has been manufactured by rolling and reliability for bolts is 99.9%.

  a) Determine bolt and member stiffnesses and stiffness ratio C.

  b) What is the factor of safety guarding against fatigue failure ?



Solution of Example 8.9

Metric grade **8.8 M14x1.5** bolt has: S_y=660 MPa, S_u=830 MPa and S_p=600 MPa.

Table 8–11

Metric Mechanical-Property Classes for Steel Bolts, Screws, and Studs*

Property Class	Size Range, Inclusive	Minimum Proof Strength,† MPa	Minimum Tensile Strength,† MPa	Minimum Yield Strength,† MPa	Material	Head Marking
4.6	M5–M36	225	400	240	Low or medium carbon	4.6
4.8	M1.6–M16	310	420	340	Low or medium carbon	4.8
5.8	M5–M24	380	520	420	Low or medium carbon	5.8
8.8	M16–M36	600	830	660	Medium carbon, Q&T	8.8
9.8	M1.6–M16	650	900	720	Medium carbon, Q&T	9.8
10.9	M5–M36	830	1040	940	Low-carbon martensite, Q&T	10.9
12.9	M1.6–M36	970	1220	1100	Alloy, Q&T	12.9

Solution of Example 8.9

Metric grade **8.8 M14x1.5** bolt has tensile area of A_t = 125 mm^2.

Table 8–1

Diameters and Areas of Coarse-Pitch and Fine-Pitch Metric Threads.*

Nominal Major Diameter *d* mm	Coarse-Pitch Series			Fine-Pitch Series		
	Pitch *p* mm	Tensile-Stress Area A_t mm^2	Minor-Diameter Area A_r mm^2	Pitch *p* mm	Tensile-Stress Area A_t mm^2	Minor-Diameter Area A_r mm^2
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980
72	6	3460	3280	2	3860	3800
80	6	4340	4140	1.5	4850	4800
90	6	5590	5360	2	6100	6020
100	6	6990	6740	2	7560	7470
110				2	9180	9080

Solution of Example 8.9

Stiffnesses of bolt and members and stiffness ratio of the joint are determined as follows. Stiffness of the bolt:

$$k_b = \frac{A_b E_b}{L} = \frac{\pi(14)^2(207000)}{4(40)} = 796.62 kN/mm$$

Based on the assumption that compression spreads out with 45° cone angle, k_m is calculated from:

$$k_m = \frac{\pi E d}{2\left[\ln \dfrac{5(L+0.5d)}{(L+2.5d)}\right]} = \frac{\pi(207000)(14)}{2\left[\ln \dfrac{5(40+7)}{(40+35)}\right]} = 3985.8\ kN/mm$$

Based on the assumption that compression spreads out with 30° cone angle, k_m is calculated from:

$$k_m = \frac{1.813 E d}{2\ln\left(\dfrac{2.885L+2.5d}{0.577L+2.5d}\right)}$$

$$k_m = \frac{1.813(207000)(14)}{2\ln\left(\dfrac{2.885(40)+2.5(14)}{0.577(40)+2.5(14)}\right)} = 2761009 N/mm$$

<u>If we use Wileman's equation</u>

$$k_m = 0.78715 E d e^{0.62873 d/L}$$

$$k_m = 0.78715(207000)(14)e^{0.62873(14/40)} = 2842660\, N/mm$$

<u>If we use Filiz's equation</u>, $B_1 = (0.1d/L)^2 = (0.1*14/40)^2 = 0.001225$ and $B_2 = 0$ (since $L_1 = L_2$), we obtain $\boldsymbol{k_m}$

$$k_m = \frac{\pi}{2} E_{eq} d e^{\left(\frac{\pi}{5} - B_1\right)(d/L)} \frac{1}{1 - B_2} \longrightarrow k_m = \frac{\pi}{2} 103500(14)e^{0.21948} = 2834708\, N/mm$$

We found four different values among which three of them yield almost the same value (3.985 MN/mm, 2.761 MN/mm, 2.842 MN/mm, 2.834 MN/mm).
To be more safe, **2.761 MN/mm** is selected for $\boldsymbol{k_m}$ for the rest of the calculations.

$$C = \frac{k_b}{k_b + k_m} = \frac{796.2}{796.2 + 2761} = 0.224$$

Solution of Example 8.9

Bolt is initially tightened to %60 of the prof load means it is preloaded with a force which is equal to 60 percent of the proofload.

$$F_p = A_t S_p = 125(600) = 75000N \longrightarrow F_i = 0.6F_p = 0.6(75000) = 45000N$$

<u>Alternating and mean components of the force on the bolt :</u>

$$F_a = C\frac{F_e}{2} = 0.224(10000) = 2240N \qquad F_m = F_i + F_a = 45000 + 2240 = 47240N$$

<u>Corresponding stresses are determined as:</u>

$$\sigma_a = \frac{F_a}{A_t} = \frac{2240}{125} = 17.92MPa \qquad \sigma_m = \frac{F_m}{A_t} = \frac{47240}{125} = 377.92MPa$$

<u>Endurance limit of the bolt is:</u>

$$S_e = k_c k_e S'_e \quad \text{and} \quad S'_e = 0.45S_u \longrightarrow S'_e = 0.45(830) = 373.5MPa$$

$$k_e = \frac{1}{K_f} = \frac{1}{3} = 0.333 \quad \text{and } k_c = 0.753 \;\; (\, for \;\; R = 99.9\%)$$

$$S_e = k_c k_e S'_e$$

$$S_e = 0.753(0.333)(373.5) = 93.75 MPa$$

It is now possible to determine factor of safety by using Modified Goodman theory of failure:

$$n = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u}} = \frac{1}{\frac{17.92}{93.75} + \frac{377.92}{830}} = 1.55$$